

## 1 Overview

The recovery of the absolute flow field from measurements of temperature and salinity along a hydrographic section is one of the fundamental problems in modern oceanography. The velocity as an explicit function of density  $\rho = R(T, S, p)$  is very sensitive to variations in density. Instead we apply an inverse method to estimate the absolute velocity by minimizing a cost function

$$\mathcal{J}(x) = \frac{1}{2}(\mathbf{m}(x) - \mathbf{d})^T W (\mathbf{m}(x) - \mathbf{d}).$$

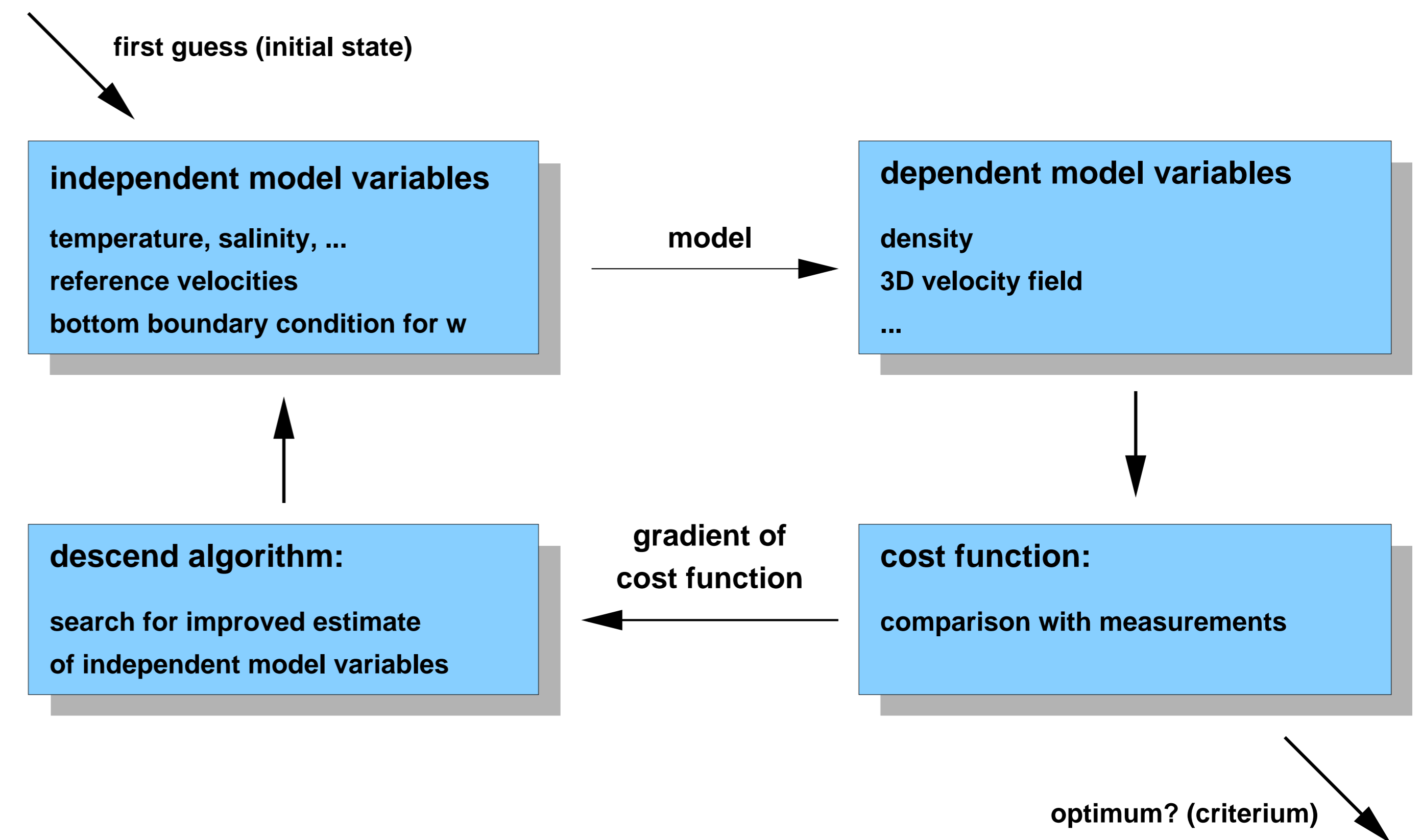
This functional is a measure of the distance between the model estimate  $\mathbf{m}(x)$  and the data  $\mathbf{d}$  as a function of the independent model variables  $x$ . The measure is defined by the *a priori* error statistics of both model and data. Simultaneously, the velocity field is constrained by various balance equations:

- geostrophy = balance of pressure gradient und coriolis force
- linear quasi-geostrophic vorticity equation
- advective balance tracer equations (temperature and salinity)
- conservation of mass

The minimization problem with non-linear equations of many degrees of freedom ( $\approx 10^4$ ) can be solved by an iterative method. The resulting solution is only complete with an estimate of the errors which can also be calculated with an iterative method.

The inverse model is augmented to include measurements of sea surface height by satellite altimetry (TOPEX/Poseidon). When referenced to an accurate geoid model, additional information can improve the estimates of the flow field substantially.

## 2 Minimization: Adjoint Method

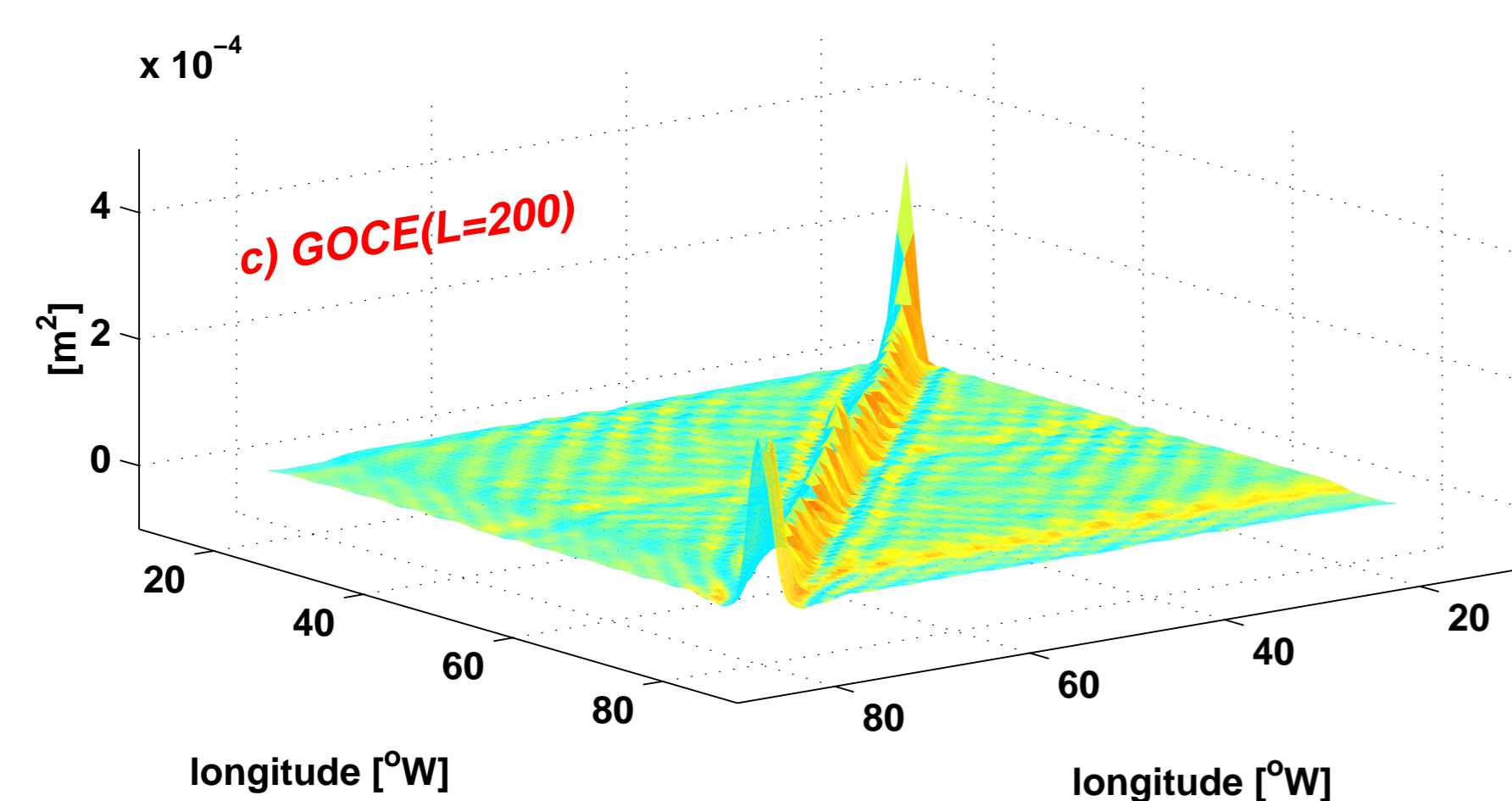
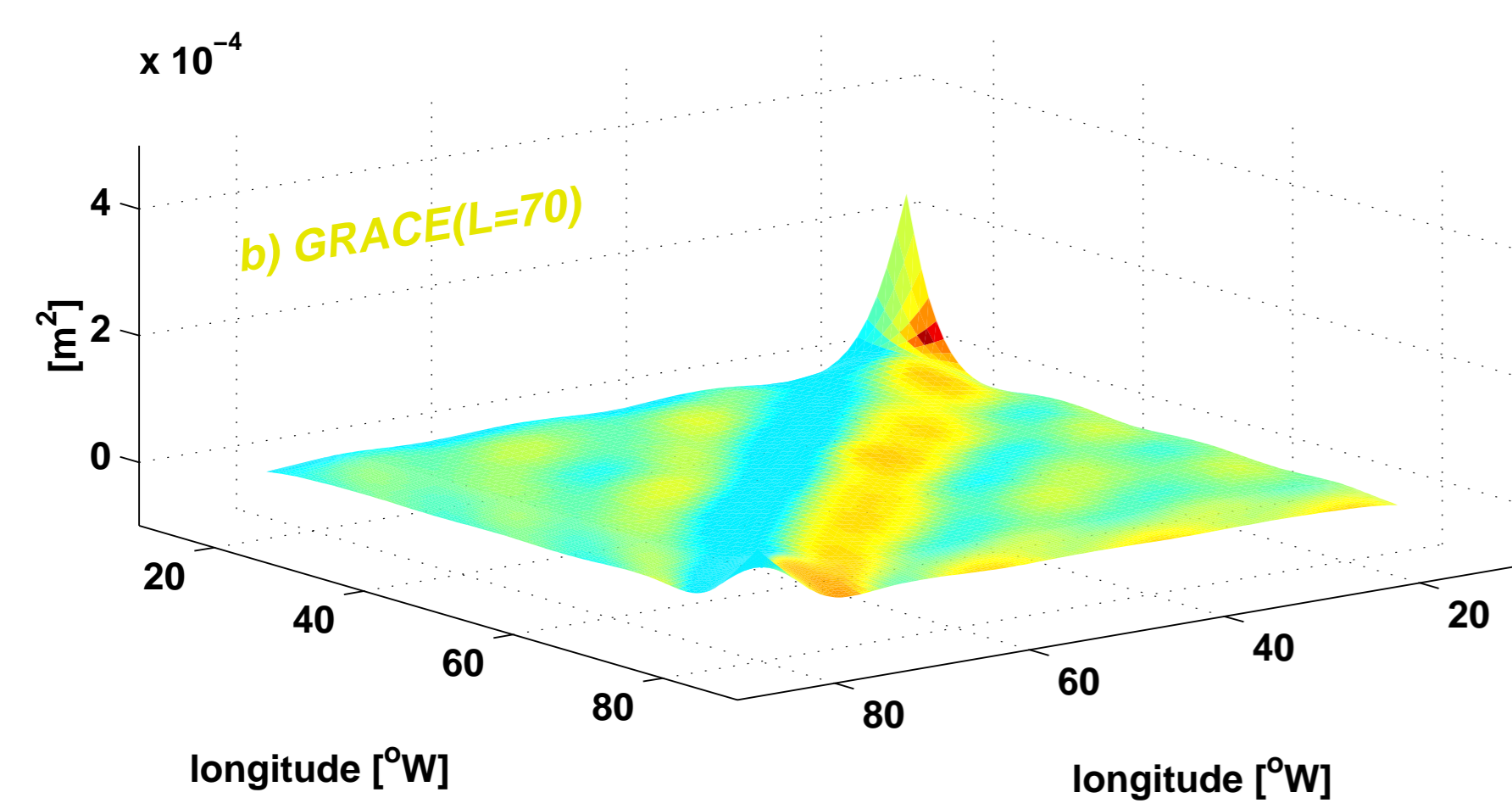
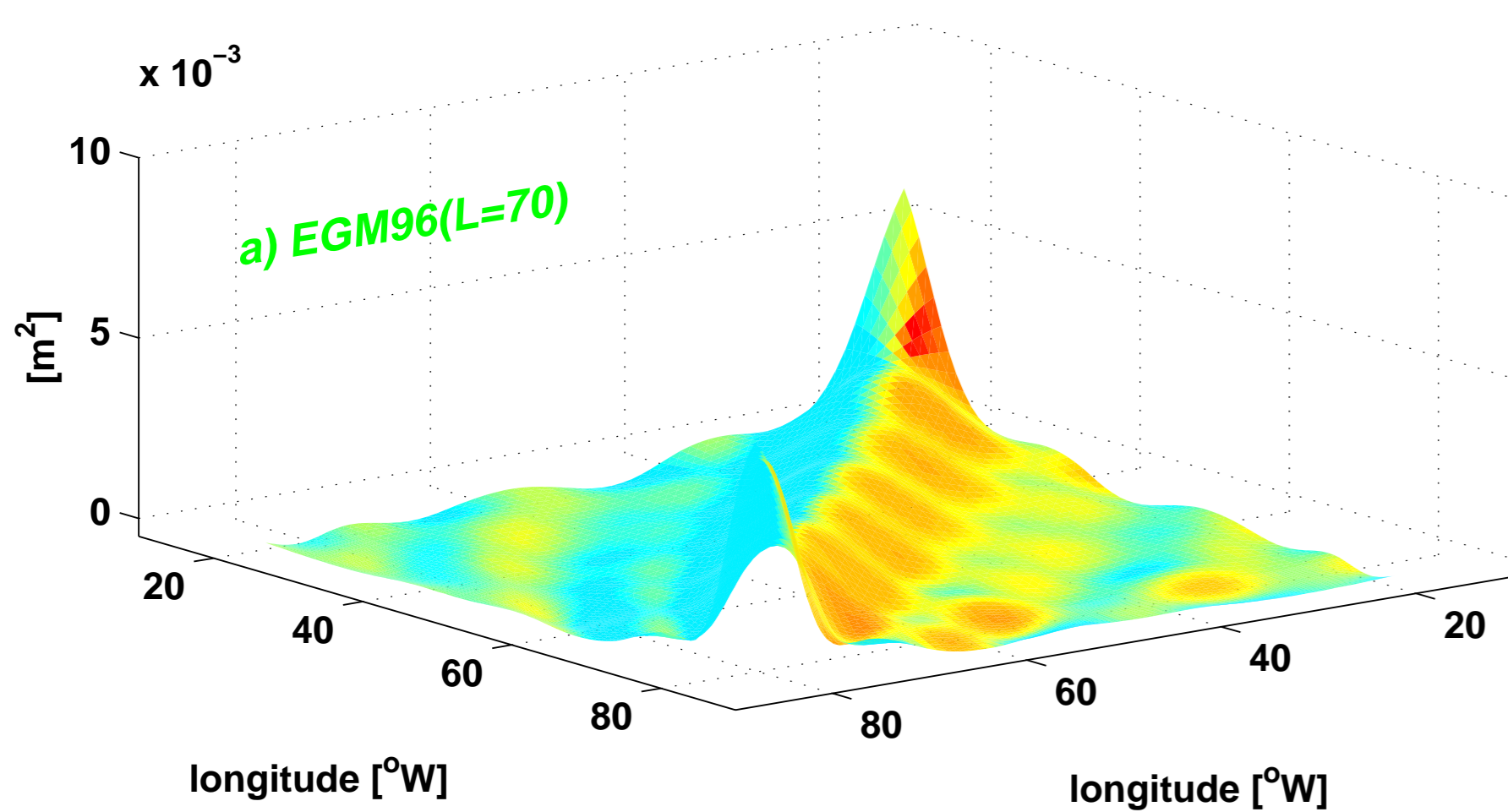


## 3 The Geoid Problem

The altimeter measures the distance between itself and the sea surface. To obtain the dynamic sea surface height, from these measurements an equipotential surface, the geoid must be subtracted. To this date, the inaccuracy even of the state-of-the-art geoid model **EGM96** limits the use of these data to time-dependent phenomena in the ocean. New dedicated satellite gravity missions will determine geoid models of high accuracy (**GRACE**) and resolution (**GOCE**).

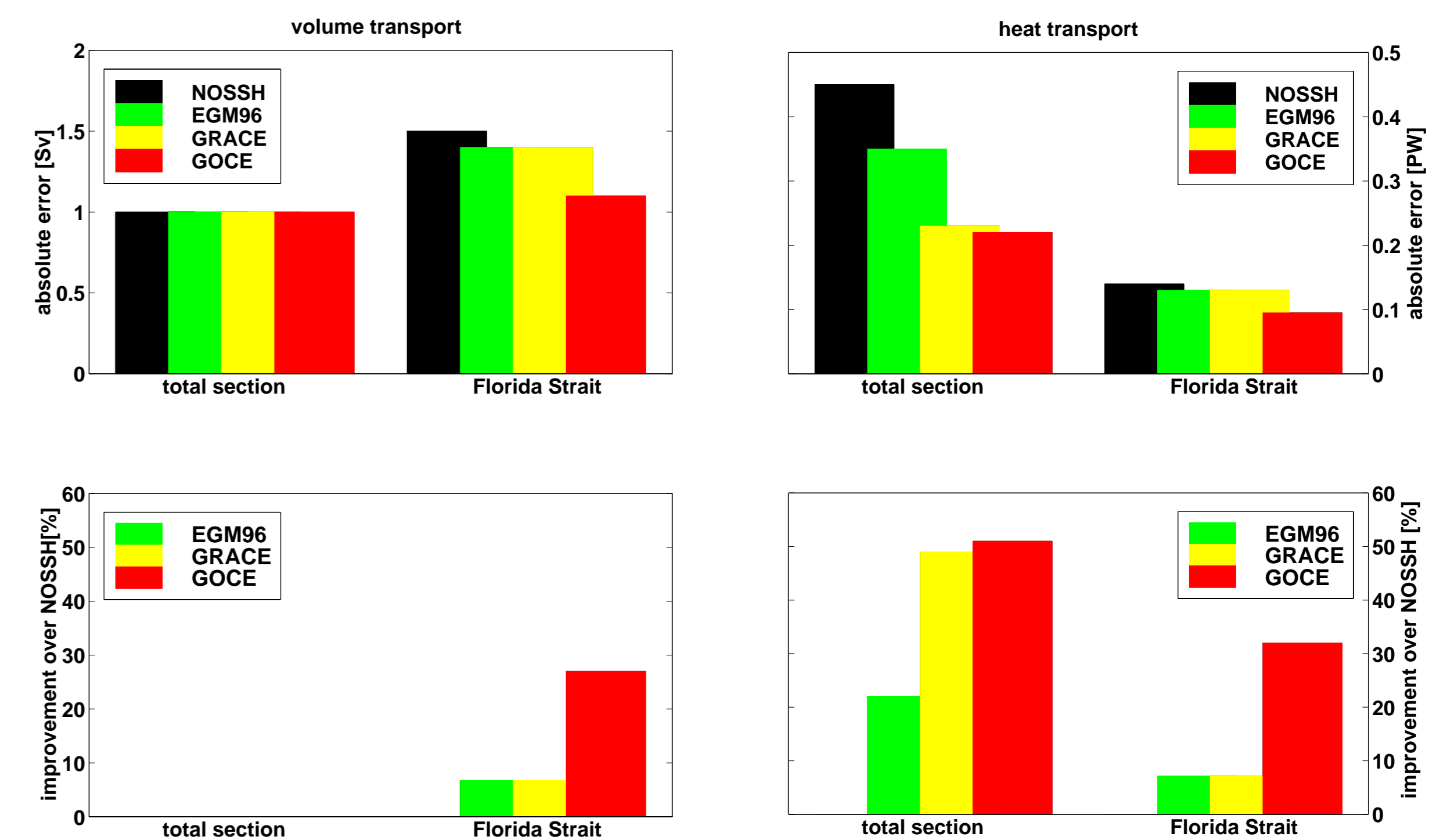
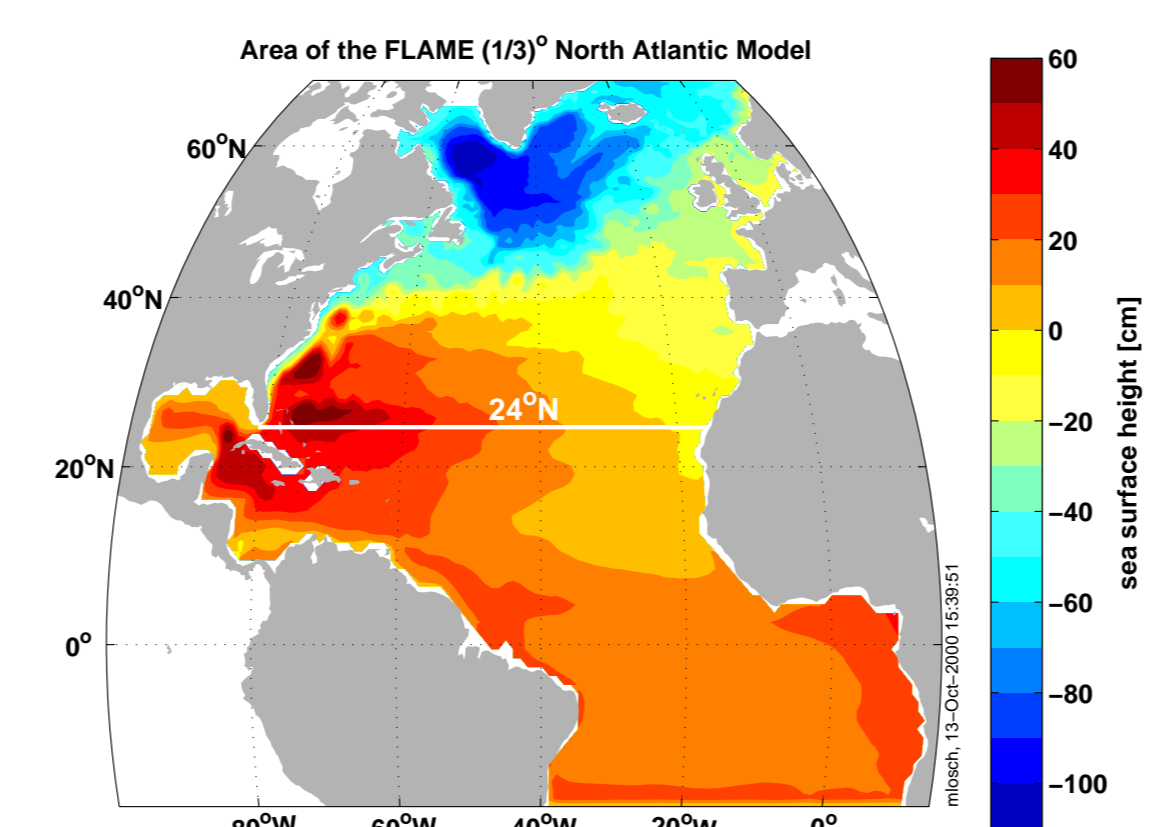
geoid	EGM96	GRACE	GOCE
resolution	290 km	290 km	100 km
mean error	10 cm	0.6 mm	6 mm

## 4 SSH error covariance matrices = geoid error covariance + measurement error



## 5 Sensitivity Experiments with Synthetic Data of the North Atlantic Model of the FLAME Group

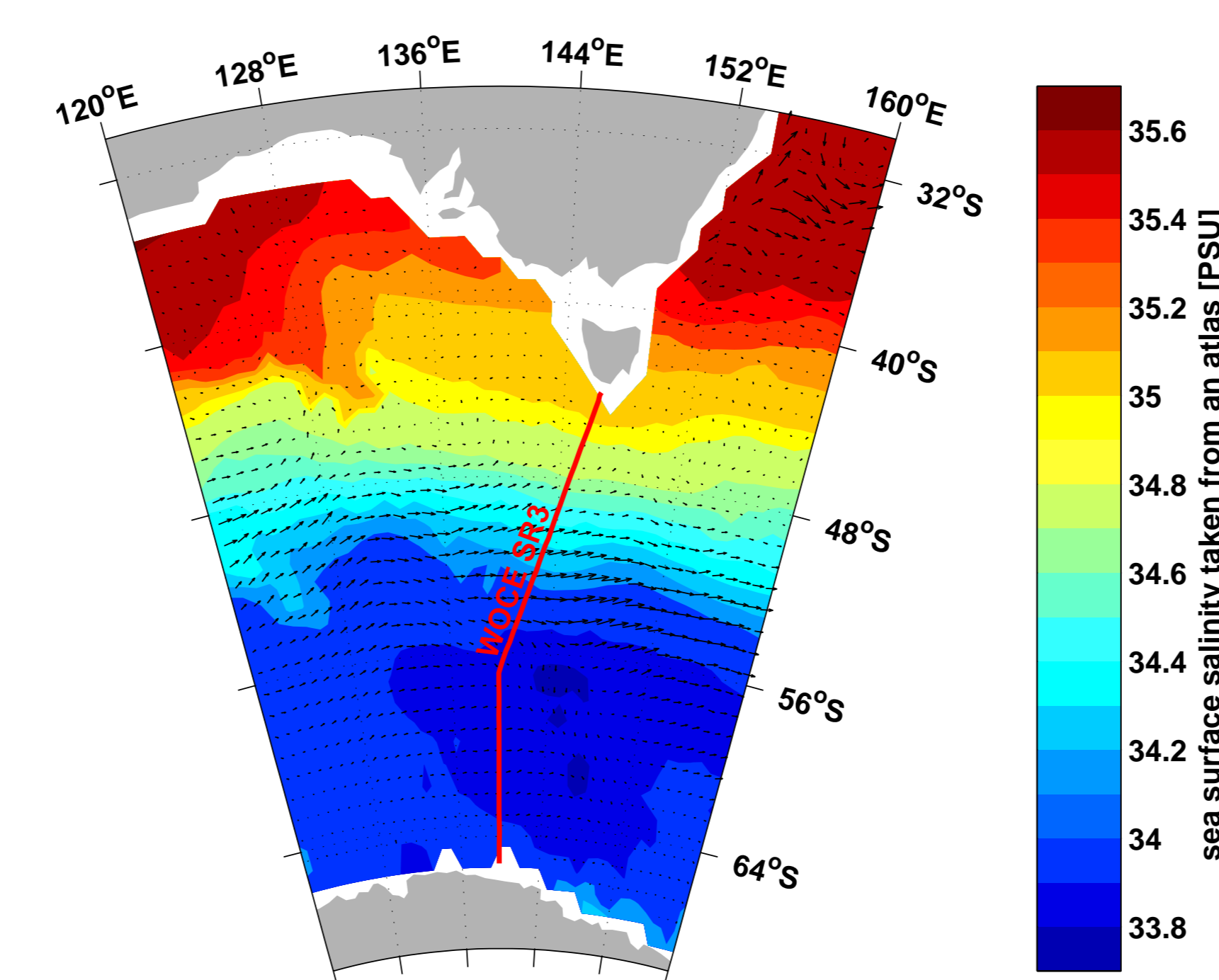
With more accurate data for dynamic sea surface height, drastic improvements of transport estimates through a hydrographic section along 24°N are possible.



## 6 WOCE SR3

The method is applied to the WOCE Southern Ocean Repeat section SR3. Only long scales of the sea surface topography are to the data (mean sea surface height SHOMCLS98.2 relative to EGM96). By the use of these data the transport estimates become more accurate by approximately 30%.

### Model Area



### Transport Estimates

	RUN: without SSH	with SSH <sup>a</sup>	Improvement
volume transport [Sv]	154 ± 13	160 ± 8	34%
heat transport [PW]	1.93 ± 0.14	2.02 ± 0.10	31%

<sup>a</sup>relative to EGM96

### Velocity Fields

