# An Empirical Model for Wind-Driven Surface Velocity using Altimeter, Wind, and Pacific Drifter Data

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## ABSTRACT

Ralph and Niiler (1999) (RN) analyzed long-term mean ageostrophic circulation measured by WOCE drifters at 15m depth in the tropical Pacific. Their best statistical model had both the amplitude of the current and its vertical scale proportional to wind speed and inversely proportional to the square root of the Coriolis parameter. The 15m ageostrophic current vectors were optimally described by an angle to the wind and a magnitude proportional to the wind speed. We have repeated this analysis with 2-day averaged (i.e. time dependent) drifter data in the Pacific Ocean, using only data that cross Topex/POSEIDON (T/P) altimeter tracks, so that along-track altimeter-derived sea surface heights (SSH) can be used to estimate and remove the time-varying geostrophic velocity component. The mean SSH was supplied from the 1998 World Ocean Atlas (Levitus, et al., 1998) (WOA98) data. The model velocities are fit to the observed 15m drifter velocities by adjusting the geostrophic smoothing scale as well as the wind model angle and scale so as to minimize the residual (datamodel) variance. The fit is done is done both before and after removing the long-term time mean from the observations.

Both along-track T/P SSH and 2-D gridded AVISO SSH (Ducet, et al., 2000) were used to estimate and remove the geostrophic velocity. The along-track SSH performs slightly better than the gridded product, reducing the variance of the observations by up to 70%. The optimal smoothing scale for computing geostrophic velocity from along-track SSH varies significantly with latitude, with a form that seems to be consistent in both hemispheres, regardless of whether the means are removed from the data. In almost all cases, the fractional variance reduction is smaller for the anomaly data than the original, which is consistent with the smaller signal-to noise ratio of the anomalies.

The best-performing wind model reduced the residual drifter velocity variance by up to 30% further, for a total variance reduction of about 80%. The angle between the wind and the 15m velocity shows marginally significant variability with latitude, but centers on the 55 degree value found by RN. Likewise, the proportionality of wind-driven current to wind speed shows some variation with latitude, while remaining within about 10% of the value chosen by RN. Winds from both the NCEP/NCAR reanalysis and the Atlas SSM/I product (Atlas, et al., 1996) were tried in the fits, and the Atlas product yielded up to 5% better [Work supported by NASA] variance reductions



Figure 2: Fractional variance reduction (variance of crosstrack drifter velocities after subtraction of estimated geostrophic velocity, normalized by the variance before subtraction) plotted vs. half the along-track length over which the surface slope was computed by a least-squares fit. The minimum for this latitude range (20-26N) occurs at 85 km, with an uncertainty of  $\pm$  5 km derived from the curvature at the minimum.



Figure 3: Histogram of residual velocities after the geostrophic fit normalized by their observed standard deviation.



Both drogued and corrected undrogued data are present in this figure. Crossing points with a time lag between altimeter and drifter larger than 3 days are not shown, and were not used in the analysis. Data in the 4 degree band around the equator were not used in the analysis.

#### **Drifter Data**

Figure 1 shows the locations of all 45000 data used in the fits. Two-day averaged drifter data were selected where their paths crossed altimeter ground tracks within 3 days of the altimeter pass. Both drogued and corrected undrogued data (Niiler, 2001, Pazan and Niiler, 2001) are used. Data in the 4 degree band around the equator were left out of the analysis. Data number and distribution varies somewhat depending on the altimeter and wind product used, because points are dropped where data are missing. Likewise, when mean velocities are removed, the data are excluded when the drifter mean has less than 60 drifter days in a 2 by 2 degree bin.

## **Altimeter Data**

The along-track T/P altimeter SSH data were used to estimate geostrophic velocity by leastsquares fitting a line to a range of altimeter points centered on the crossing points. When time means were included in the fit, the time-mean SSH was derived from the WOA98 data using a reference level of 2000 m. The number of points used in the fit was varied from 7 to 51, producing a varying averaging scale from 37 km to 310 km. Figure 2 shows an example of fractional variance reduction vs half the smoothing interval. Fractional variance reduction means variance of crosstrack drifter velocities after subtraction of estimated geostrophic velocity divided by the variance before subtraction. The minimum for this latitude range  $(20^{\circ}-26^{\circ}N)$  occurs at 85 km, with an uncertainty of  $\pm$  5 km derived from the curvature at the

Figure 3 shows a histogram of residual velocities after the geostrophic fit, normalized by their observed standard deviation. The distribution is very roughly Gaussian. Only 89 out of 45000 data were dropped as outliers after the fits, and their exclusion had insignificant effects on the optimal parameters, although the amount of variance fit increased by up to 3% when outliers were excluded.

Figure 4 shows the number of good points in each latitude bin, showing the relative lack of data at high latitudes. The bins centered on  $\pm 47$  degrees are judged to be near the minimum number of data required for reliable estimates, based on cross-validation studies done in bands with larger numbers of observations.





#### **Geostrophic Velocity from Altimeter SSH**

Optimal least-square fit length vs. latitude is shown in Figure 5. These only apply to the geostrophic velocity estimates from along-track Topex/POSEIDON altimeter height, which was converted from slope to velocity by the geostrophic relation at the latitude of the observation. Since only cross-track geostrophic velocity could be estimated (except at altimeter rossover points), the drifter data were projected to give only the component of velocity normal to the altimeter track. The results are symmetric around he equator, and are consistent within error bars  $(\pm 5 \text{ km})$  across both total and anomaly datasets. The 2-D gridded altimeter product was treated similarly, but was available on a 1/4 degree grid and fitting lengths were limited to 1 or 2 grid points on either side of the crossing. The fit generally worked better at the longer lengths. The variance reduction at the optimal fit ength is shown in figure 6. The along-track fits are significantly better than the 2-D gridded product in all cases, with the near-equatorial data showing the largest difference. These could perhaps have been reduced by including more grid points, and averaging over a longer interval, since the along-track results found long averaging intervals near the equator, but issues of anisotropy may have become important as well.



Figure 5: Optimal least-square fit length vs. latitude. These all apply to the geostrophic velocity estimates from along-track Topex/POSEIDON altimeter height, which was converted from slope to velocity by the geostrophic relation at the latitude of the observation. The smoothing scale that produced the minimum residual variance is plotted. The figure has 6 curves: 'N. Hemi' and 'S. Hemi' refer to Northern and Southern hemisphere, respectively; the sign of the southern latitudes have been changed to overlay the plots. The 'dmn' notation identifies the results from using only SSH anomalies and drifter velocity anomalies. The 'noley' notation identifies results from using SSH anomalies and full drifter velocities.

> Figure 4 a,b (Left): Number of good points in each latitude bin. 4 curves are plotted: 'tpx ncep' uses along-track Topex/POSEIDON altimeter height anomalies and winds from the NCEP reanalysis; 'tpx atlas' uses along-track altimeter height anomalies added to WOA98 dynamic topography and Atlas winds; '2d ncep' uses the AVISO gridded surface height anomaly product and ncep winds; and '2d atlas' uses the gridded anomalies with atlas winds. The winds have no effect on the geostrophic fit, but can change the number of data points available depending on the distribution of missing points. Plot "a" uses observed winds and currents in the analysis, with the mean sea surface height (SSH) supplied by dynamic height estimates from WOA98 (Levitus), while "b" uses only SSH anomalies with respect to a 7-year mean, and has removed mean winds and currents calculated from the surface drifter dataset. There are fewer points in the demeaned dataset because data are omitted when there are too few drifter days to determine a good mean.

Figure 8 (Right): Wind model residual fraction vs. latitude. Residual fraction is the variance after the velocity estimated from the wind model has been removed normalized by the variance before the wind model was used. In both cases the geostrophic velocities had been removed. Part 'a' shows results for total velocity, winds, and SSH, 'b' shows results for only anomalies. Each plot ('a' and 'b') has 4 curves as in Figure 4: 'tpx ncep' uses along-track Topex/POSEIDON altimeter height; 'tpx atlas' uses along-track altimeter height and Atlas winds; '2d ncep' uses the AVISO gridded surface height anomaly product and ncep winds; and '2d atlas' uses the gridded anomalies with atlas winds.



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Figure 6: Geostrophic variance reduction vs latitude at the optimum fitting scale from Figure 5. Part "a" shows the total dataset, while "b" shows the de-meaned data. Note that the choice of wind product only affects these curves by changing the number of observation

The wind model used by Ralph and Niiler (1999) assumes linearly decreasing stress over an Ekman layer H. The ageostrophic (wind-forced) velocity is written in complex notation ( $U_a = u + iv$ ,  $\tau = \tau_x + i\tau_y$ ):  $U_a = \frac{e^{i\theta}}{H} \frac{\tau}{\rho f}$ 

Introducing the friction velocity  $u_*^2 = |\tau|/\rho$ , and  $\hat{\tau} = \tau/|\tau|$ , this can be

$$U_a = \frac{e^{i\theta}}{H} \frac{\hat{\tau} u_*^2}{f}$$
(2)

In the case of a constant austauch coefficient  $A_{\nu}$ , the Ekman layer

 $H = u_*^2 (2/fA_v)^{1/2}$ 

If  $A_v$  is proportional to  $u_*^2$ , then this becomes

 $H = \beta u_* f^{-1/2}$ Which is Ekman's original hypothesis, and (2) becomes:

 $U_a = \beta e^{i\theta} \hat{\tau} u_* f^{-1/2}$ 

Nijler (1999) found optimal values of  $\theta = 55^{\circ}$  and  $\beta = 0.065$  by fitting to binned mean ageostrophic currents in the tropi-





## Wind Model

The wind model optimization took place after the geostrophic velocity had been removed, so that the variance reduction figures are relative to the residual variance after the geostrophic fit. The two fits did not seem to interact in a few trials, and were kept separate for simplicity. Figure 7 shows an example of the surface of residual variance fraction in the RN wind model angle and scaling parameter space using all data in the North Pacific between 20N and 26N. In this example, cross-track geostrophic velocities had been removed, so this optimization is only for the velocity component in the cross-track direction. The Atlas wind product was

Figure 8 shows the wind model residual fraction vs. latitude, for both total and demeaned datasets. Along-track altimeter data outperform the 2-D product, in general, and Atlas winds outperform the NCEP/NCAR reanalysis wind product.

Figure 9 shows the angle which produced the minimum residual variance when using the RN model. The scatter in the curves can be taken as a measure of the error bars, although the results for the Atlas winds and along-track altimetry have the best error bars based on the curvature at the minimum, about  $\pm 5$  degrees in angle. There is an apparant trend in the curves, although the high latitude values are determined by relatively few data, and the equatorial values suffer from poorer geostrophic velocity removal. It is interesting that the wind model, which can reduce the residual variance by roughly another 20%. better geostrophic fits lead to better performance of the wind model, suggesting that residual geostrophic velocities act as noise to reduce the performance of the wind model fit.

Figure 10 shows wind model scaling factor vs. latitude for these same data. The scaling factor multiplies the 0.065 drag factor used in the reference RN model. Aside from greate scatter at the equator and high latitudes, the RN value (1.0 in these units) seems to be preferred in general.





nodel angle vs. latitude. This shows the angle which produced the minimum residual variance when using the RN model. The two panels and the 4 curves are as in figure 8.



Figure 10: Wind model scaling factor vs. latitude. This shows the scaling factor which produced the minimum residual variance when using the RN model. The scaling factor multiplies the 0.065 drag factor used in the reference RN model. The two panels and the 4 curves are as in figure 8.

#### **Conclusions:**

Altimeter-derived geostrophic velocity explains about half of the drifter velocity variance, and the smoothing scale varies strongly with latitude.

Removal of geostrophic velocity significantly improves the performance of the

RN values for wind model parameters seem to apply in the time-dependent case, although there seems to be a trend with latitude, which may have to do with unmodeled parameters, such as mixing layer depth.

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