Surface and internal tides from bottom pressures and altimetry

1. Introduction

Internal tides are thought be play an important role in mixing the deep ocean (Munk & Wunsch 1998). The global tidal analysis of Egbert & Ray (2000) has shown that tidal energy is dissipated in the deep ocean, presumably by conversion to the internal tide. Ray & Mitchum (1996) identified the internal tide off Hawaii in TOPEX/Poseidon measurements.

Recent theoretical work has reexamined the internal tide generation process (St. Laurent & Garrett 2001, Llewellyn Smith & Young 2002; see Figure 1). However, comparisons with measurements has been limited so far.

In this study surface and internal tides are computed from bottom pressure and altimetric data at locations in the South Atlantic. The Smith & Sandwell bathymetry for the region is shown in Figure 2, along with the locations of the bottom pressure recorders (BPR).

Figure 1 Tidal conversion

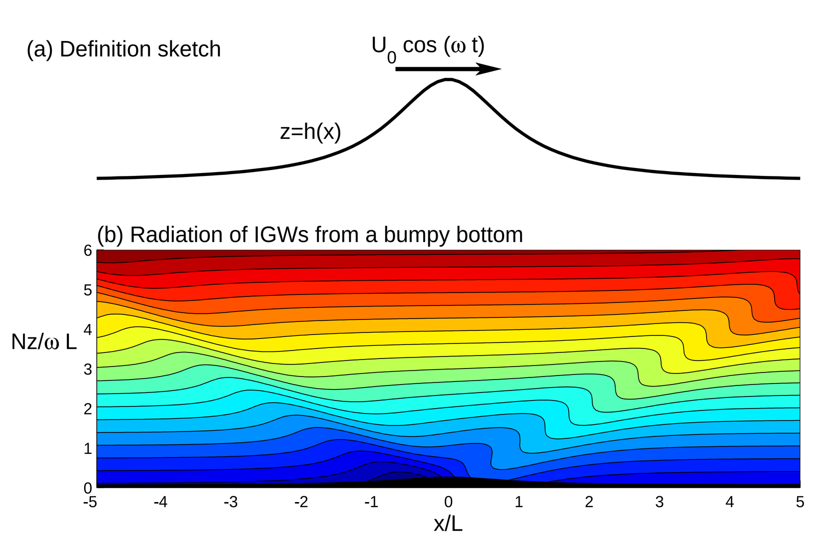
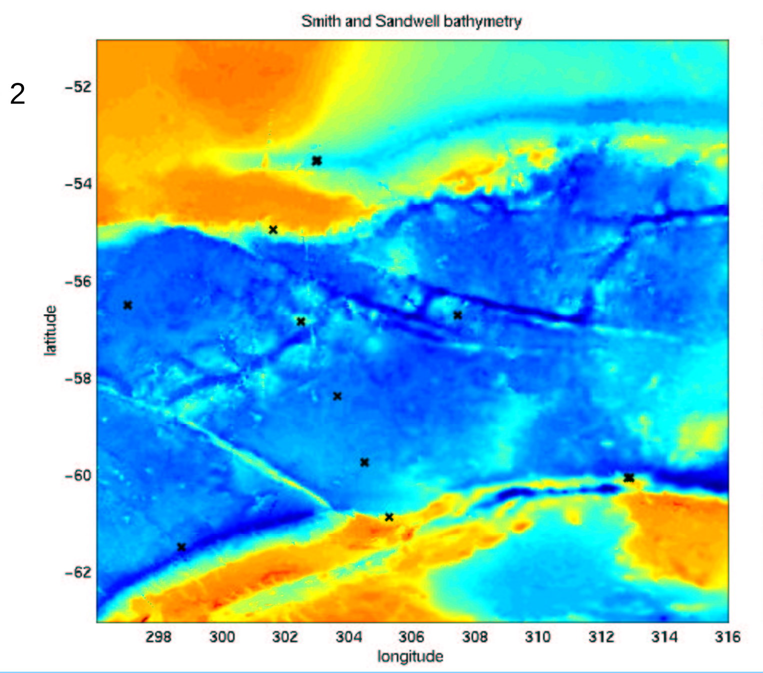


Figure 2



2. TOPEX measurements

The OSU Tidal Inversion Software, or OTIS, (Egbert & Erofeeva 2002) is a freely available tidal inversion scheme. Local tidal models using linear or nonlinear dynamics can be fitted to TOPEX/Poseidon "no-tide"data. Here, OTIS is used to interpolate TOPEX measurements of the M_2 tide onto the locations of the BPRs. The dynamical model is ignored by weighting it by a factor of 10^{-9}. Tidal records are produced at the BPR locations. Figure 3 shows the domain and in yellow the data points used (red points are irrelevant; blue points are not used).

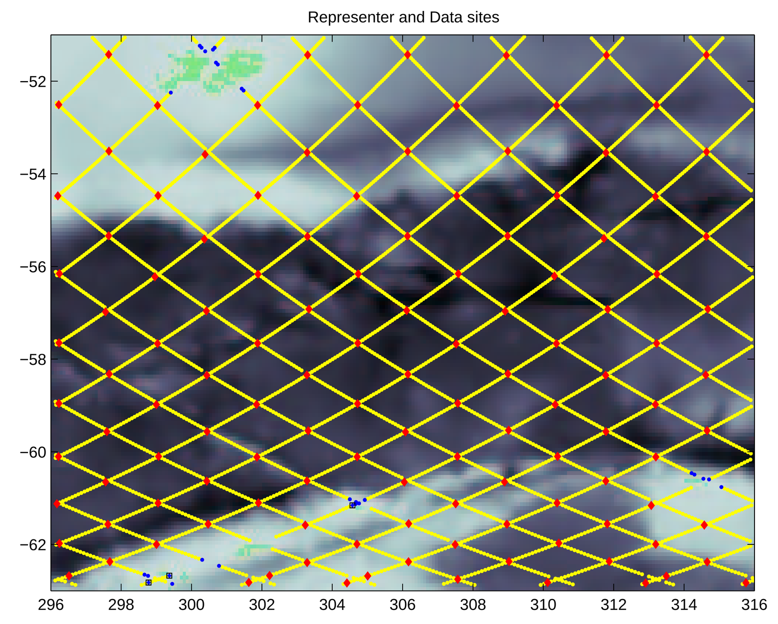


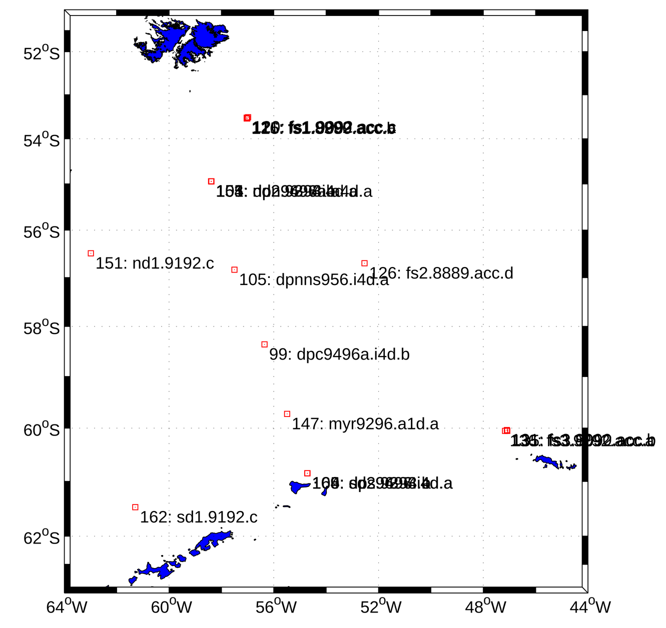
Figure 3: TOPEX/Poseidon groundtracks

3. GLOUP measurements

GLOUP, the Global Undersea Pressure data set, is the global data bank for ocean bottom pressure. GLOUP is maintained by Proudman Oceanographic Laboratory (`http://www.pol.ac.uk/psmslh/gloup/gloup.html`). The data have been quality controlled and put into 'TASK-2000' form. 279 time series are available spanning the period 1969–1999, with high-frequency data sampled at either 15 minute or hourly intervals.

A number of POL deployments were carried out in Drake passage and the South Atlantic during the 1990s. These time series overlap with the TOPEX/Poseidon mission and are close together spatially. The area of the South Atlantic between 63 and 51S and 64 and 44W was hence selected for this study. The GLOUP BPR locations and names are shown in Figure 4.

Figure 4: GLOUP BPR locations



5. Surface and internal tides

Assuming that the tidal motion can be represented by the barotropic mode $\hat{p}_0$ and the first baroclinic mode $\hat{p}_1$ gives

$$p(0) = a_0\hat{p}_0(0)+a_1\hat{p}_1(0), \qquad p(-H) = a_0\hat{p}_0(-H)+a_1\hat{p}_1(-H).$$

We have $\hat{p}_0(0) = \hat{p}_0(-H)$, and the vertical normal mode calculation gives the ratio $r = \hat{p}_1(0)/\hat{p}_1(-H)$. The bottom pressure is known, while the surface pressure is given by $\rho_0 g h_0$, where h_0 is the TOPEX sea-surface measurement. Hence

$$\begin{pmatrix} 1 & r \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} \rho g h_0 \\ p_b \end{pmatrix},$$

which may be solved to give

$$a_0 = \frac{\rho g h_0 - r p_b}{1-r}, \qquad a_1 = \frac{-\rho g h_0 + p_b}{1-r}.$$

This calculation was carried out for the M_2 tide, which was obtained using the T_Tide Harmonic Analysis Toolbox (`http://www2.ocgy.ubc.ca/fich/#T_Tide`). Figure 6 gives the resulting tidal amplitudes and phases with error bars.

Error bars were calculated using standard deviations of 2 cm for the TOPEX data (Egbert & Ray 2000 suggest 1 cm; however there is also an error due to extrapolation from the TOPEX groundtracks onto the BPR locations which is unknown – Monte Carlo simulations from the OTIS software suggest the total error is less than 2 cm), variances for the BPR data calculated from the other pressure gauges on the moorings, and variances for r and ρ_0 calculated from WOD2001 data close to the mooring.

Figure 7: tidal amplitudes and phases

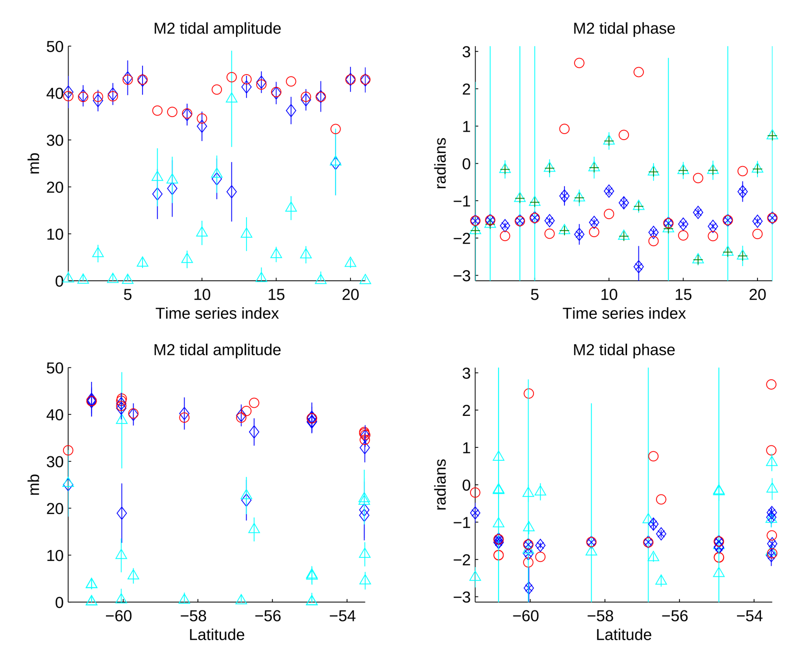


Figure 5: vertical normal modes

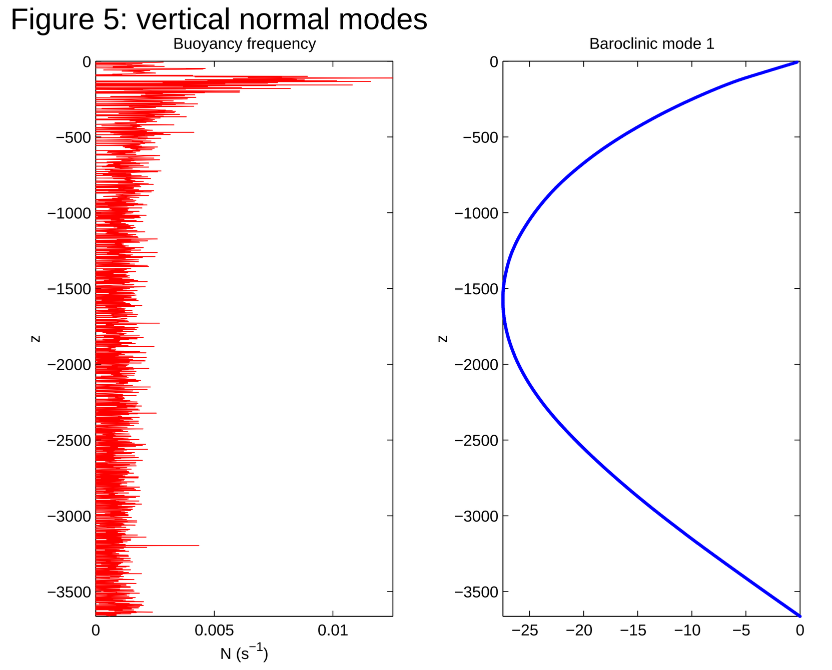
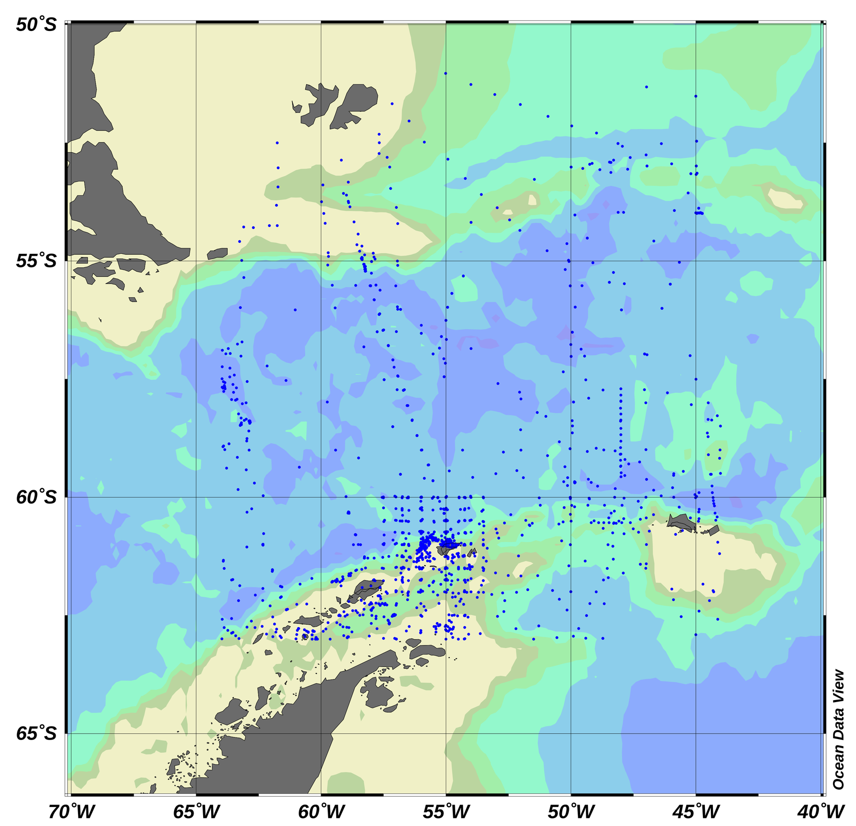


Figure 6: WOD2001 station locations



4. Vertical normal modes

The linearized equations of motion for inviscid, non-diffusive, stratified flow with rotation can be solved using vertical normal modes in the long-wave, or hydrostatic, limit (Gill 1982, § 6.11). The governing equations are then

$$\begin{aligned} u_t - fv + p_x &= 0, \\ v_t + fu + p_y &= 0, \\ p_z &= b, \\ u_x + v_y + w_z &= 0, \\ b_t + wN^2 &= 0. \end{aligned}$$

The Boussinesq approximation has been made here. In addition, b is the buoyancy, and the pressure has been rescaled. The quantity $N^2(z)$ is the buoyancy frequency.

These equations possess separable solutions of the form

$$w = \hat{h}(z)\tilde{w}(x,y,t), \qquad p = \hat{p}(z)\tilde{\eta}(x,y,t),$$

where

$$\frac{\mathrm{d}^2\hat{h}}{\mathrm{d}z^2} + \frac{N^2}{c_e^2}\hat{h} = 0.$$

An approximate solution for mode zero (the barotropic mode) is $\hat{h}_0 = z + H$, with $c_0^2 = gH$ (H is the depth of the fluid). For the baroclinic modes, the rigid-lid approximation is appropriate, and the boundary conditions become $\hat{h} = 0$ at $z = 0$ and $z = -H$. The surface displacement can then be found from the relation $\hat{h} = (\rho_0 g)^{-1}\hat{p}$ at $z = 0$.

Vertical normal modes are calculated for all WOD2001 profiles (`http://www.nodc.noaa.gov/OC5/WOD01/atlas_01.html`) within 5 degrees of BPR locations whose depth is within $H/10$ of depth of the closest profile to the BPR. The quantity $r \equiv \hat{p}_0/\hat{p}_b$ has been averaged from these results. When the data are not available, r is computed from the appropriate square in the Levitus 1994 dataset. The same procedure is applied to find the surface density ρ_0. Figure shows the profiles of buoyancy frequency and $\hat{h}$ at GLOUP station myr9296.a1d (index 147) using the closest WOD2001 profile.

6. Conclusions

The amplitudes of the surface and internal tides have been calculated from TOPEX/Poseidon and BPR data in the Southern Atlantic, assuming that the tidal energy is contained in the barotropic and first baroclinic modes. Initial results suggest it is possible to identify the baroclinic component.

Results are sensitive to the form of the vertical normal modes, which have been calculated from WOD2001 data. It would be useful to account for seasonal variability in these measurements. Another issue is the validity of using 'local' vertical normal modes when the bottom topography is not actually flat.

References

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