Multi-Mission Crossover Analysis

Forschungsgruppe
Satellitengeodäsie

Wolfgang Bosch
Deutsches Geodätisches Forschungsinstitut (DGFI), München
bosch@dgfi.badw.de

Why?

The relative calibration of contemporary altimeter systems is crucial for sea level mapping and monitoring with good spatial and temporal resolution. The large number of nearly simultaneous dual-satellite crossover differences performs a dense network with high redundancy to obtain a reliable estimate of the radial errors of all altimeter systems operating at the same time.

How?

A discrete crossover analysis is developed (see below) and applied to all crossover differences observed with at most 2 days time delay. This ensures that the crossover differences are not corrupted by sea level variations but reflect only radial errors of the altimeter systems.

Discrete crossover analysis

The crossover differences $d' = \left[\cdots, \Delta x_{ij}, \cdots\right]$ are modelled by the radial errors x_i and x_j of the two passes intersecting at a crossover location. In order to obtain a certain degree of continuity or smoothness of the errors, consecutive differences are considered in addition and - together with the crossover differences - minimized by the weighted least squares approach:

$$\begin{aligned} \Delta x_{ij} + v_{ij} &= x_i - x_j & \Rightarrow \quad d + v_x &= A\,x \\ v_{i,i+1} &= x_i - x_{i+1} & \Rightarrow \quad v_D &= Dx \end{aligned} \qquad \min_{k'x=0} \left\| \begin{bmatrix} D \\ A \end{bmatrix} x = \begin{bmatrix} 0 \\ d \end{bmatrix} \right\|_P^2$$

The combined system (right hand) has a rank defect of 1 (because the sum of all columns of A and D gives a null vector) which is solved by a single constraint $k'x = 0$ applied to all errors x_i or any subset of these errors. Here, the sum of all TOPEX/Poseidon errors was forced to be zero.

Weighting

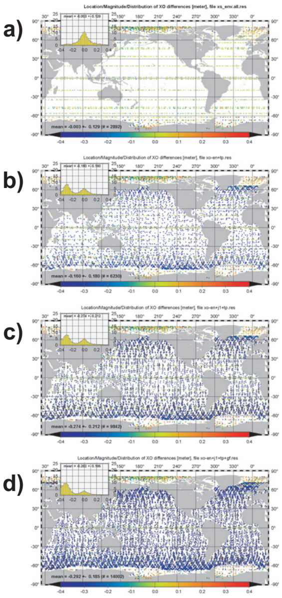
With growing time difference Δt, both „observation", Δx and x_i - x_{i+1} are down weighted - using different half-weight-width. Additional weights for the crossover differences are derived from their standard deviation and by cosφ compensating the increasing number of crossovers at high latitude.

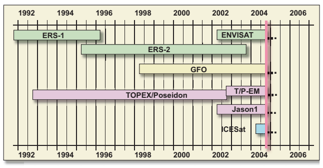
Numeric

The structure of normals for the discrete crossover analysis

$$Nx = AP_x d \quad \text{with} \quad N = D'P_d D + A'P_x A + kk'$$

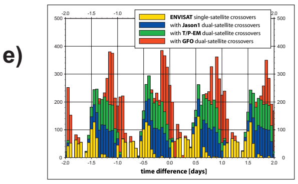
is tridiagonal (due to the first part $D'P_dD$) plus sparse (due to $A'P_xA$) – if the constraint $k'x = 0$ is applied to a single error only. However, the size of the system is large (twice the number of crossovers). The normals are solved by an iterative „Conjugate Gradient Projection" algorithm - adapted such that the repeated multiplications with the tridiagonal and the sparse part of N are treated separately. This makes the computations fast (~5 min for 80000 unknowns) and minimizes storage requirements.





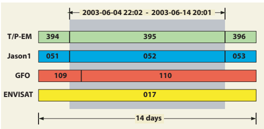
Dual-satellite crossover ...

with a time difference Δt < 2 days provide a much more homogeneous spatial and temporal distribution as single-satellite crossovers. This is shown left for ENVISAT. a) ENVISAT single-satellite crossovers b) densification by dual-satellite crossovers between ENVISAT and TOPEX/Poseidon. c) Further densification by dual-satellite crossovers with Jason1 d) All crossovers that ENVISAT performs with itself, with TOPEX/Poseidon, Jason1 and GFO. e) Temporal densification of single-satellite crossovers by dual-satellite crossover events.



Test runs for ENVISAT, TOPEX/Poseidon, Jason1, and GFO

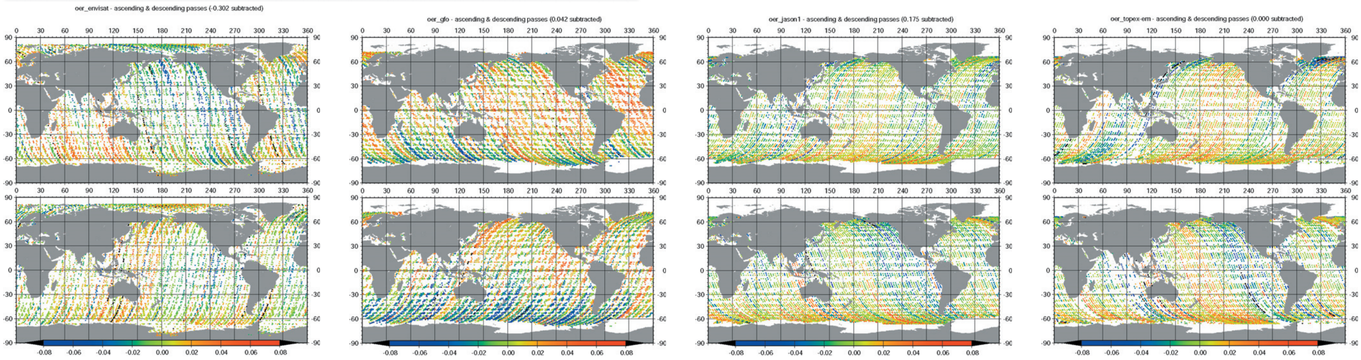
were performed for a few 10 day (T/P repeat) periods with two day overlap to neighbouring periods. Typically, the combination of all four altimeter missions leads to some 45000 crossovers with Δt < 2 days. A crossover statistic - before and after analysis - shows: relative „range biases" between different altimeter systems are captured and the rms values decrease - above all for dual-satellite crossovers.



	ENVISAT	Jason1	T/P-EM	GFO
ENVISAT **14316**	SXO: 2982 0.00 ±0.13 0.00 ±0.13	AA+DD. 3631 -0.47 ±0.07 0.00 ±0.06	AA+DD: 3523 -0.33 ±0.08 0.01 ±0.07	AD+DA: 4180 -0.29 ±0.09 0.00 ± 0.07
Jason1 **17705**		SXO: 3638 0.00 ±0.06 0.00 ±0.06	AD+DA: 5113 -0.18 ±0.07 0.00 ±0.06	AA+DD: 5323 -0.14 ±0.08 0.00 ±0.06
T/P-EM **16813**			SXO: 3253 0.00 ±0.08 0.00 ±0.07	AA+DD: 4924 0.04 ±0.08 0.00 ±0.07
GFO **18233**				SXO: 3806 0.01 ±0.07 0.00 ±0.07

Estimated radial errors

of ascending (top row) and descending (bottom row) passes of all four altimeter systems (see below). From left to right: ENVISAT, GFO, Jason1, TOPEX/Poseidon. The „range biases" were subtracted to exhibit the spatial variation of errors. Beside single passes with outstanding errors there are regional pattern with similar error characteristics.



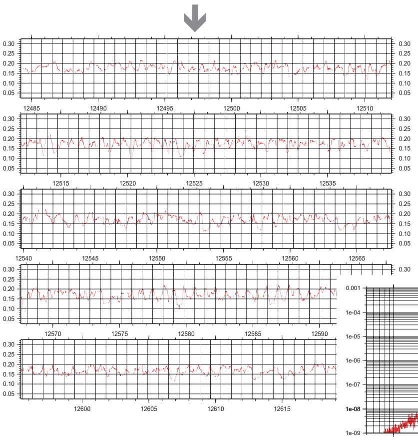
Relative range biases and centre-of-origin shifts

TOPEX/Poseidon was taken as „reference mission" (the sum of all T/P errors was forced to be zero) such that the radial errors of the other altimeter systems carry - on average - the relative range biases. A post processing of the radial errors was applied to estimate these range biases along with relative centre-of-origin shifts that explain any systematic large scale pattern. For cycle T/P cycle 395 GFO and ENVISAT show significant centre-of-origin shifts of more than 1 cm for the z and y component respectively. Note, range bias" and Δz-shifts are highly correlated due to the ocean dominance on the southern hemisphere.

mission	T/P-EM	Jason1	GFO	ENVISAT
σ_0	±0.026	±0.018	±0.027	±0.030
relative range bias Δr	-0.003	0.173	0.045	-0.301
Δx-shift	-0.005	-0.003	0.001	0.002
Δy-shift	-0.003	0.001	-0.005	0.012
Δz-shift	-0.009	-0.008	-0.015	-0.004
number of error components	19821	21318	21974	16879

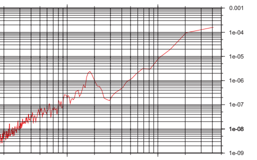
Conclusions

The discrete crossover analysis is a simple, straight forward, numerically somewhat ambitious approach to use the high redundancy provided by the many dual-satellite crossovers that can be performed between all contemporary altimeter systems. It allows a reliable estimate of the radial errors of all altimeter systems and automatically captures the range biases, relative to a reference mission.



Radial errors

Left hand, the discrete Jason1 radial errors (m) are shown as a function of time (JD2000.0). The nearly continous error distribution - not pre-defined by any functional model - allows to estimate the spectral power (as shown below).



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