Lamont-Doherty
Earth Observatory
of Columbia University

E.10: SMALL-SCALE AND SHORT-TERM VARIABILITY IN SEA SURFACE HEIGHT AND EFFECTIVE DATA ERROR

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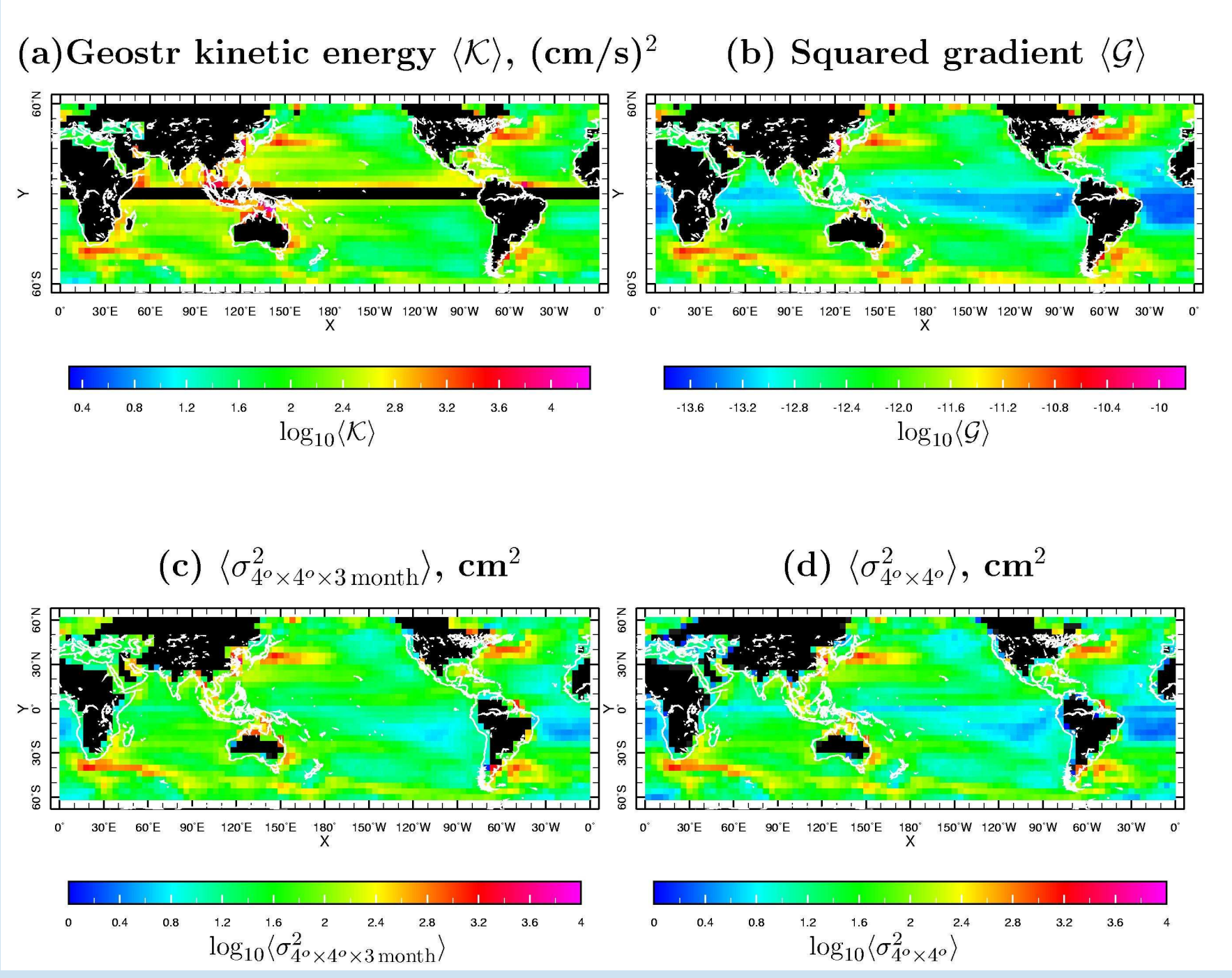
1. INTRODUCTION

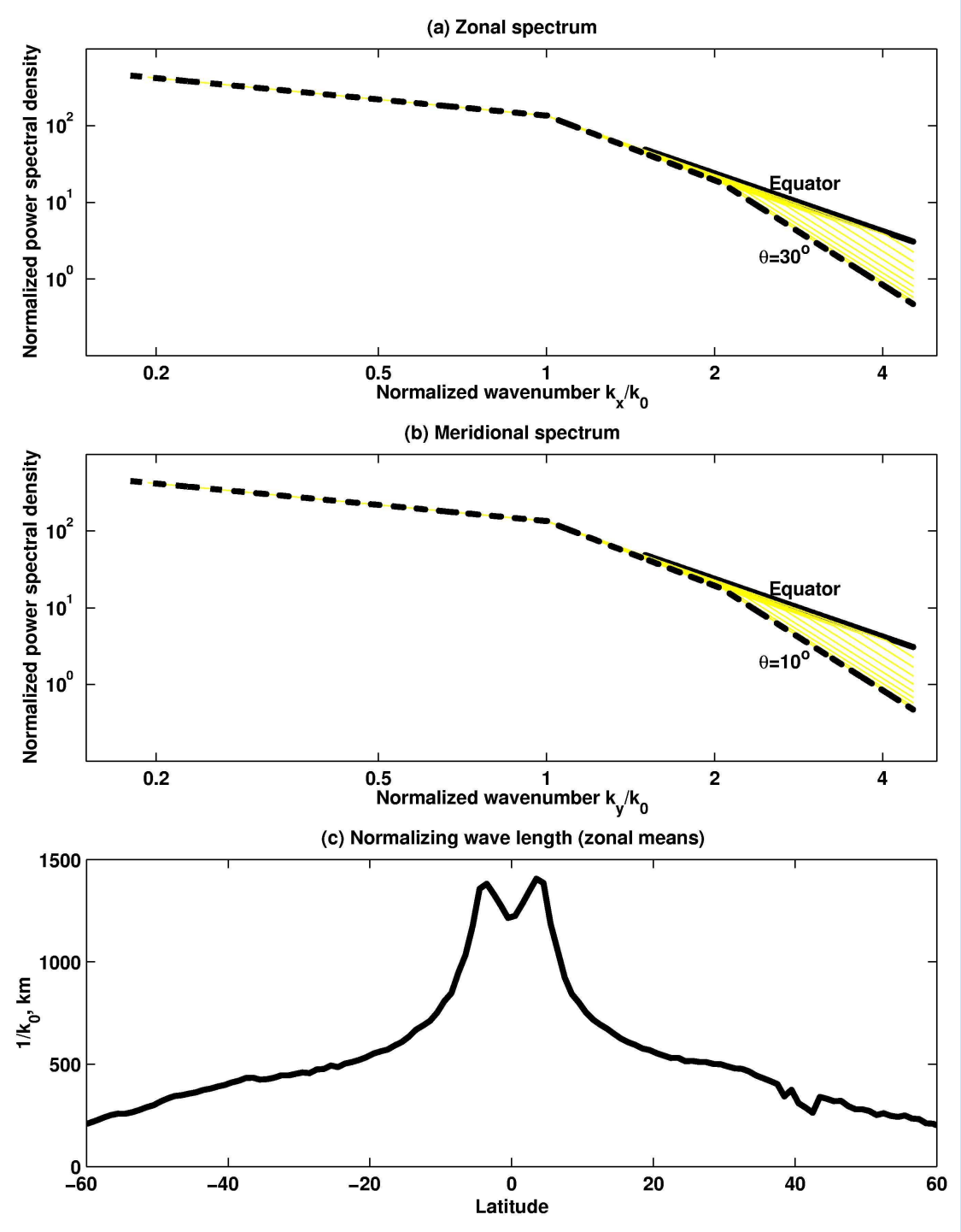
Data assimilation procedures interpret observed data as if they could be expressed in terms of the averages over model grid box areas. In reality, however, observations are either point-wise values (in cases of in situ data) or averages over certain footprints (in cases of satellite data). Therefore the difference between observations and model values ought to reflect the influence of the small-scale variability of the observed physical field, because this variability is getting averaged differently by the model grid and by the observational system. This difference turns out to be a major contribution to the effective data error and needs to be taken into account in data assimilation procedures. Multi-satellite missions to date have resulted in satellite altimetry fields of unprecedented resolution which, in turn, make it possible for us to obtain detailed descriptions of small-scale and short-term variability of sea surface height. Data error models suitable for use in data assimilation procedures are being developed. They are verified by comparing satellite altimetry analyses with in situ (tide gauge) data.

Data used: Multimission altimetry analyses (Cheney et al. 1994, Ducet et al. 2000 products, DUACS gridded products);
Tide gauge data from University of Hawaii;
Sea surface height from POCM 4C ¼ degree resolution model (Tokmakian and Challenor, 1999).

2. SMALL-SCALE VARIABILITY AND EDDY KINETIC ENERGY

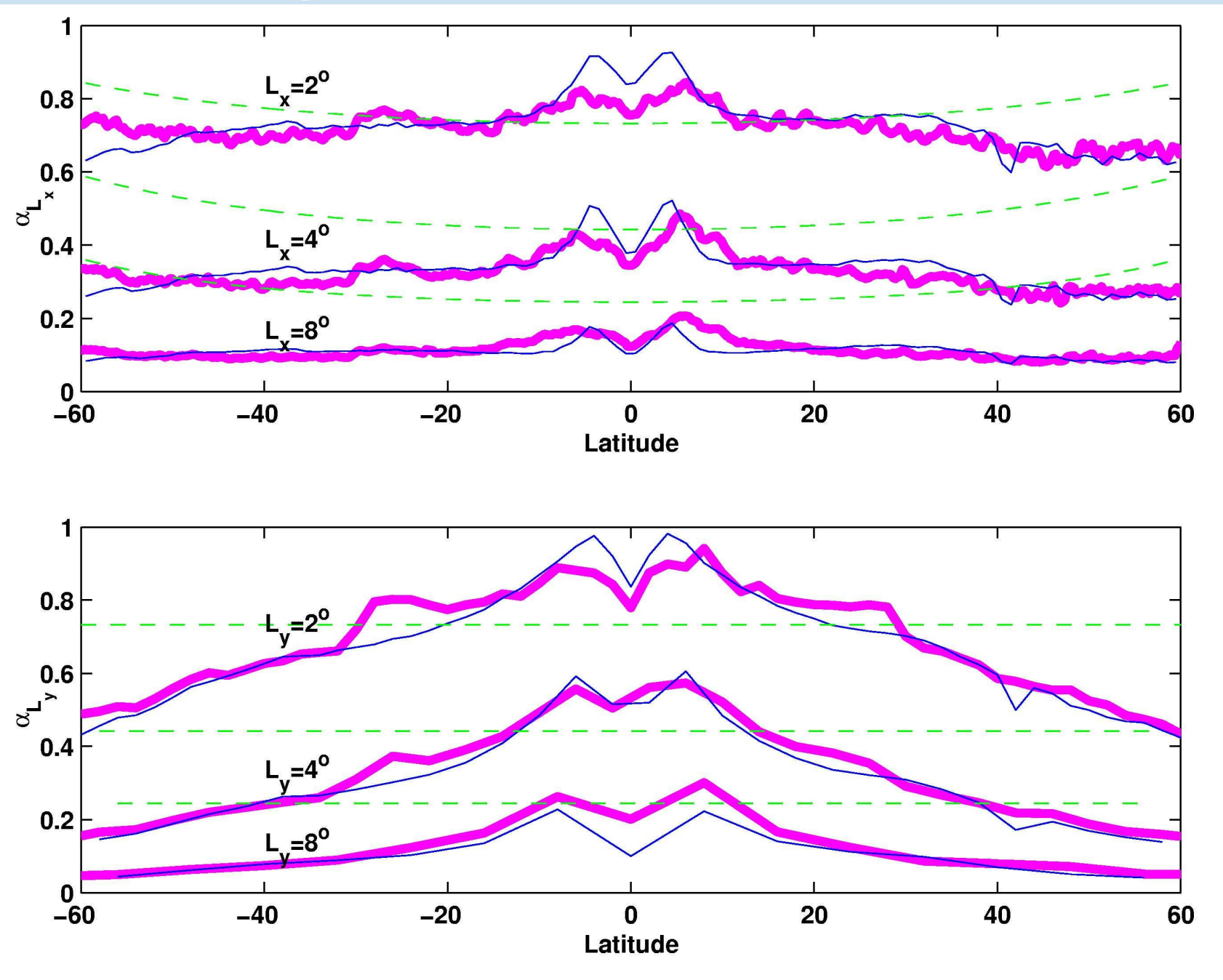


Connection between surface geostrophic kinetic energy and small-scale variability (left panel) in sea surface height: $\langle\sigma^2\rangle = C(f/g)^2 \langle K\rangle$, where $C = \alpha (L_x^2 + L_y^2)/6$, and α depends on the wavenumber power spectrum of the sea surface height. Parameter α shows how small differences in sea surface height scale to the L_xXL_y box. Stammer et al. (1997) midlatitudinal and tropical spectral approximations are spliced together for the use in this work (right).



3. FORMALISM VALIDATION

Zonal averages of α for one-dimensional variances



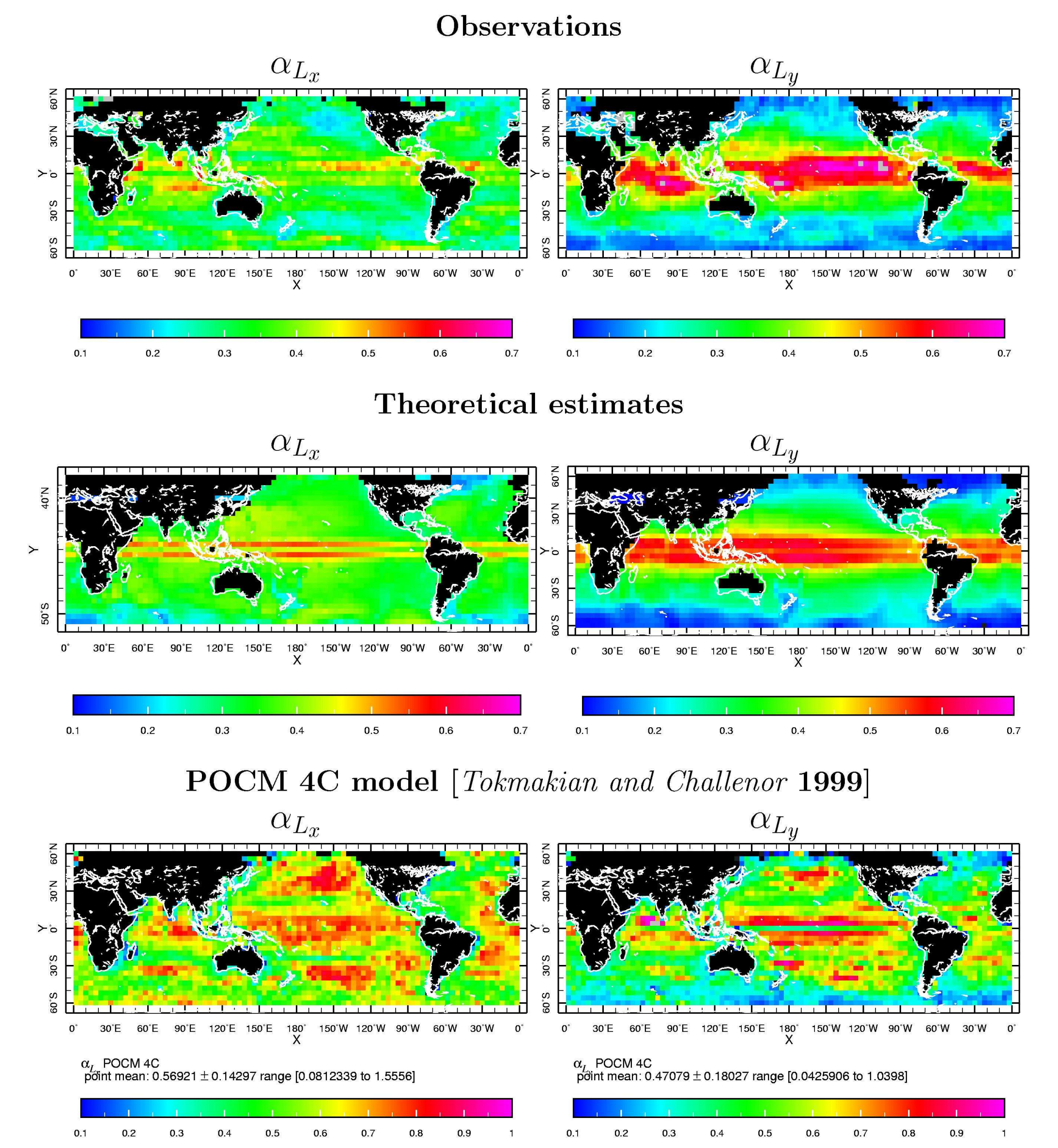
Observations (magenta)

Theoretical values for wavenumber spectral approximations:

Stammer (1997) – blue

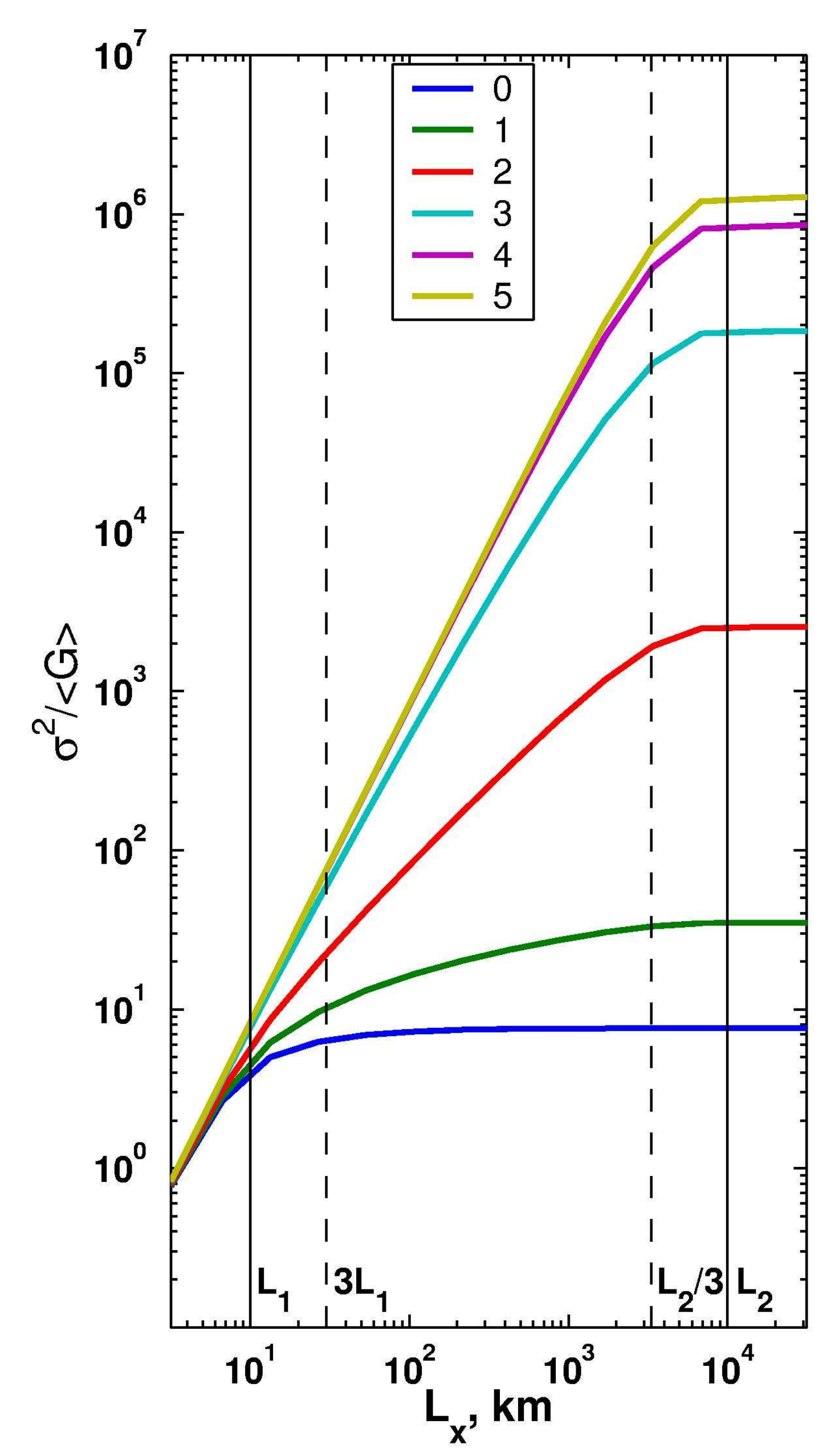
Zang and Wunsch (2001) - green

Spatial patterns of one-dimensional coefficients



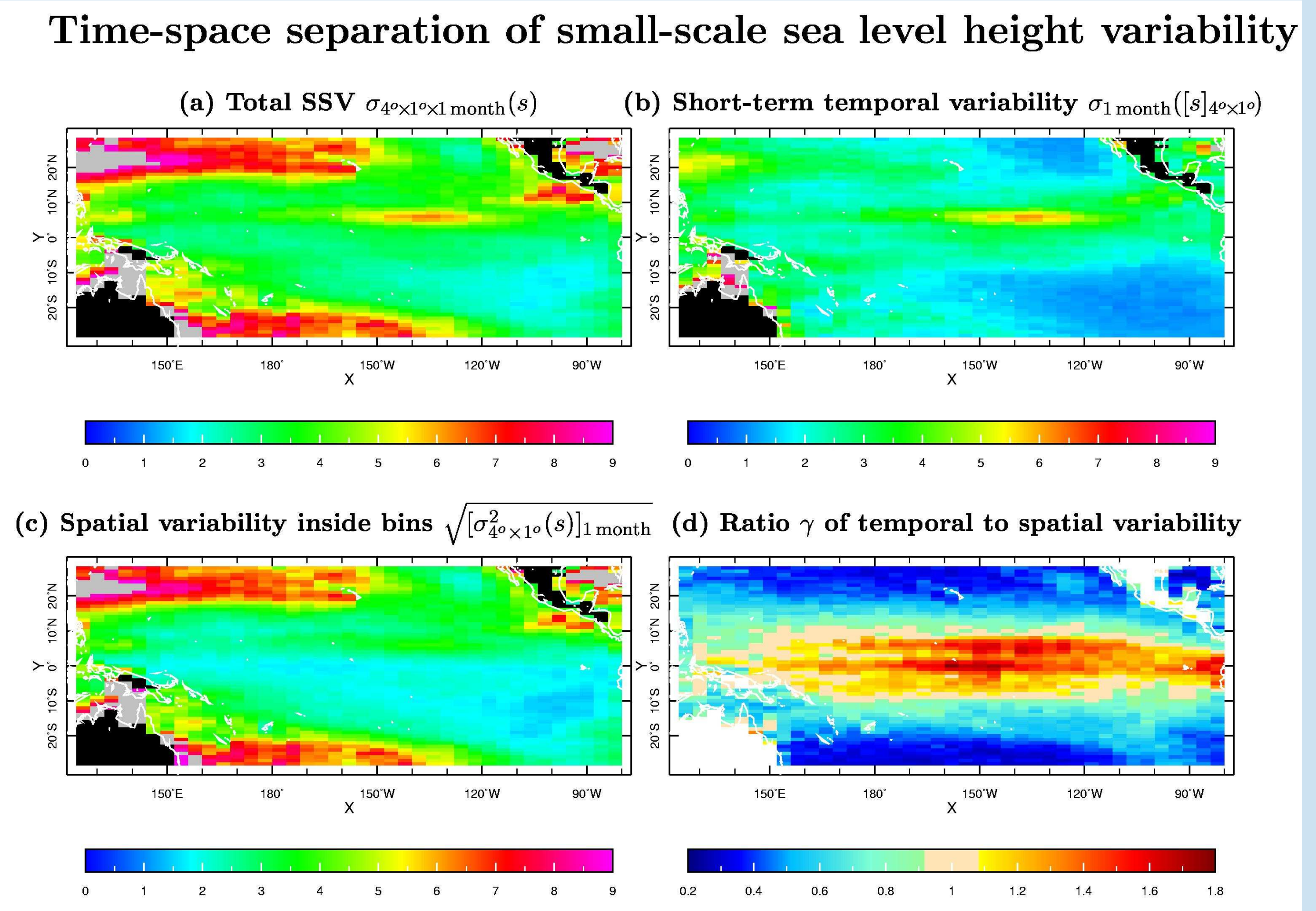
4. SCALING FOR POWER-LAW SPECTRAL FORMS:

$$P(k)=A/k^n$$

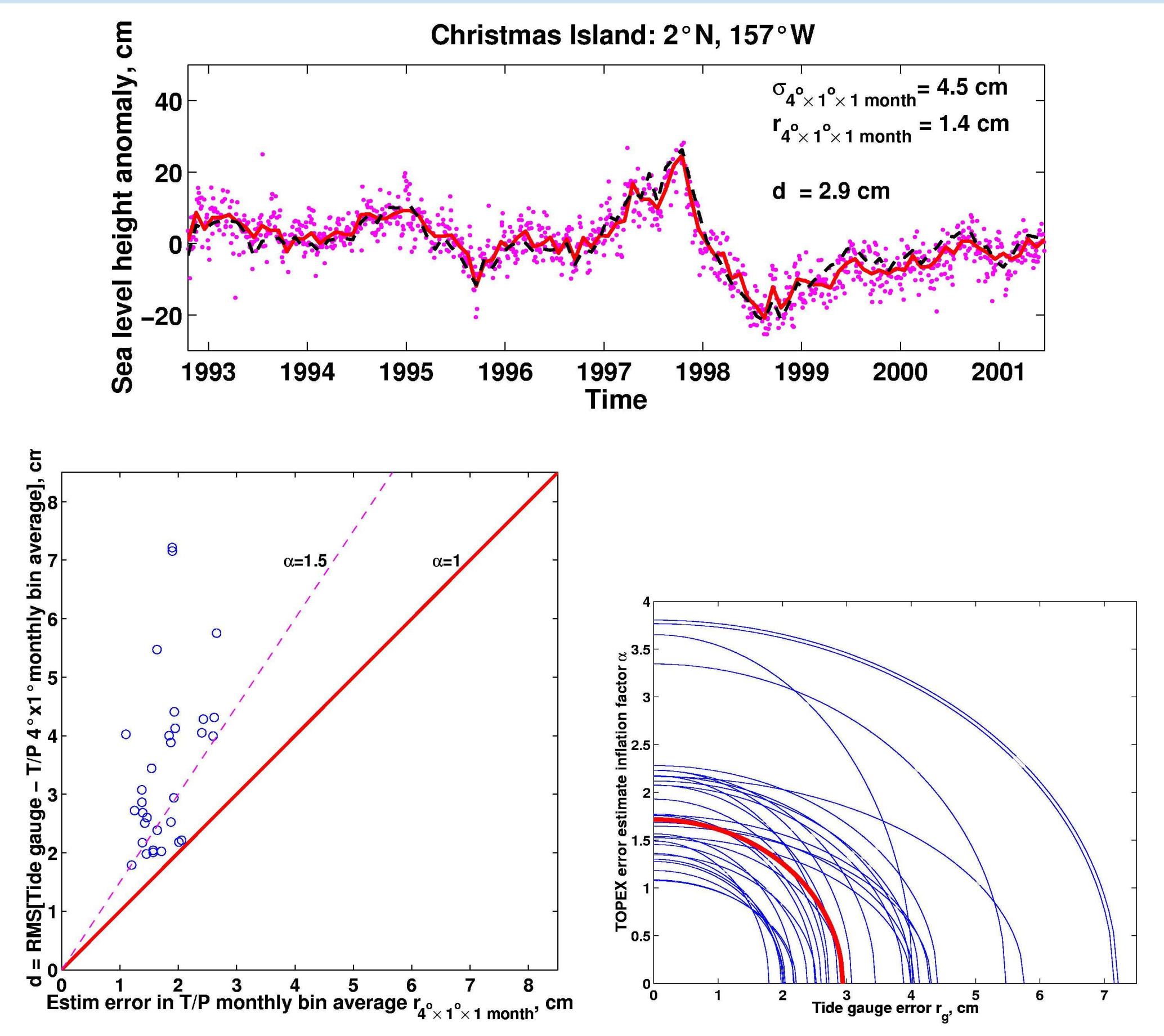


As n varies from 0 to infinity scaling of the ratio between variability under scale L and squared sea surface height gradient changes between L^0 and L^2. Scaling power stays at 2 for n values larger than 4.

5. CONNECTION BETWEEN SPACE AND TIME

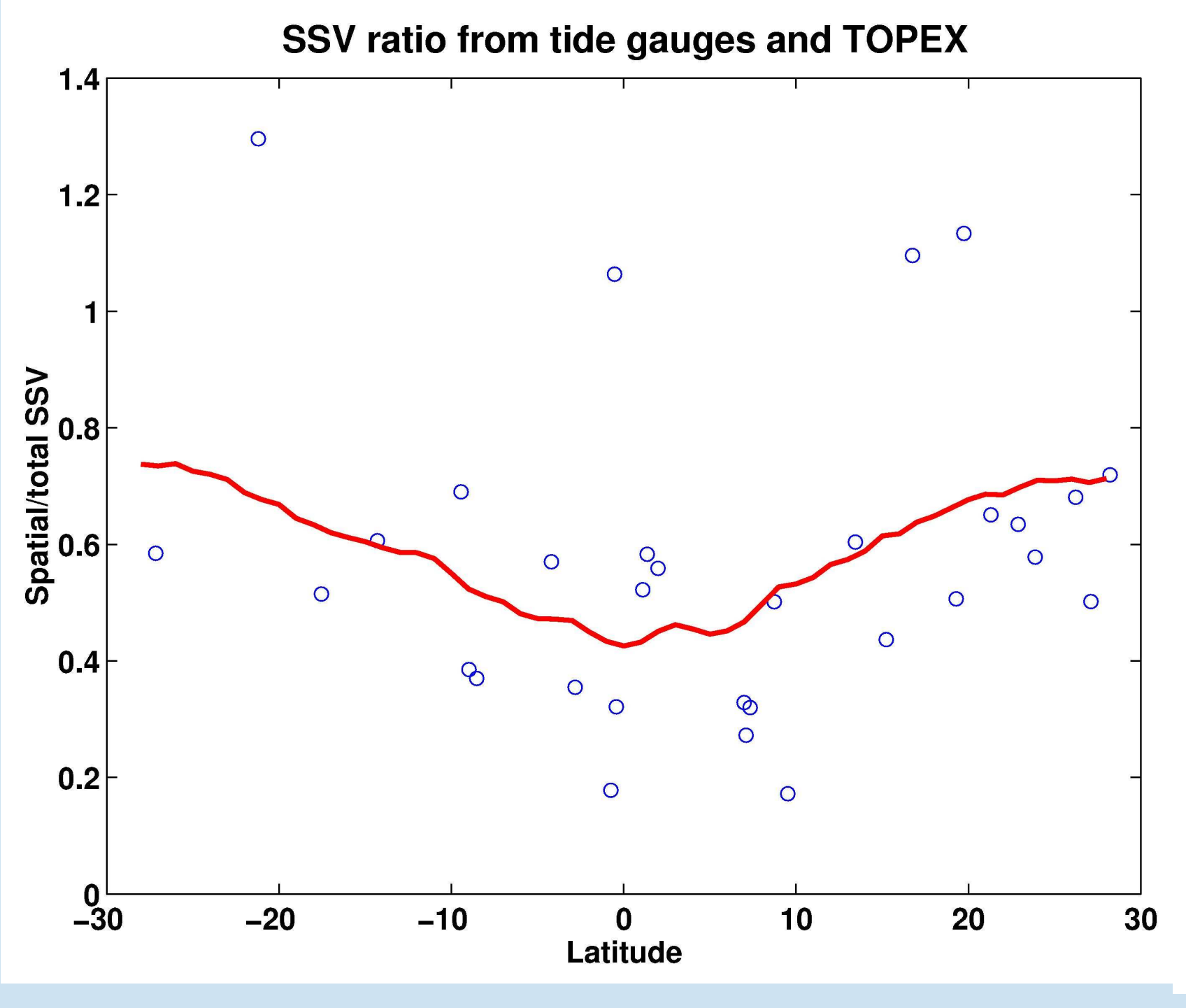


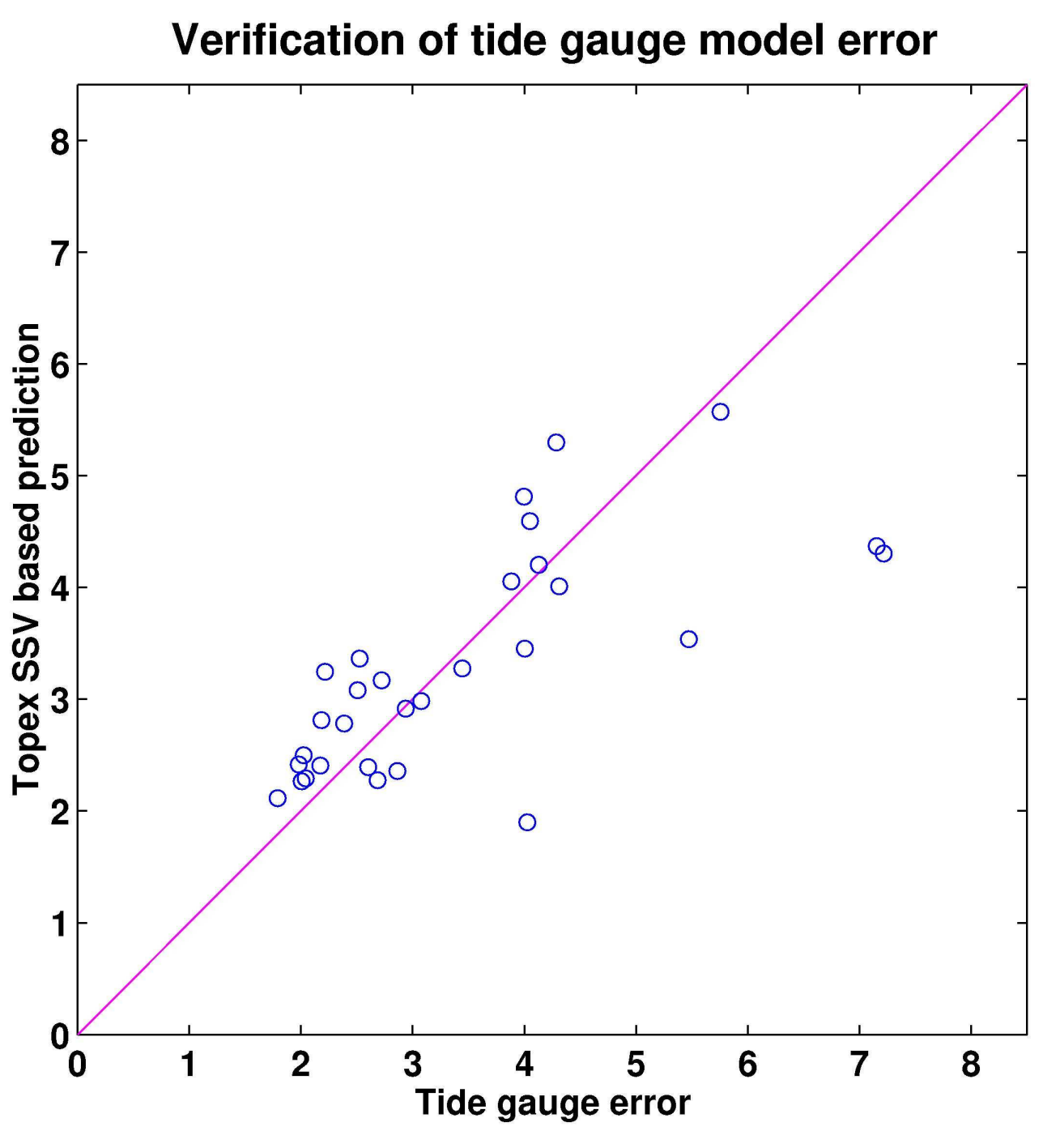
6. ERROR MODELS FOR ALTIMETRY AND TIDE GAUGES



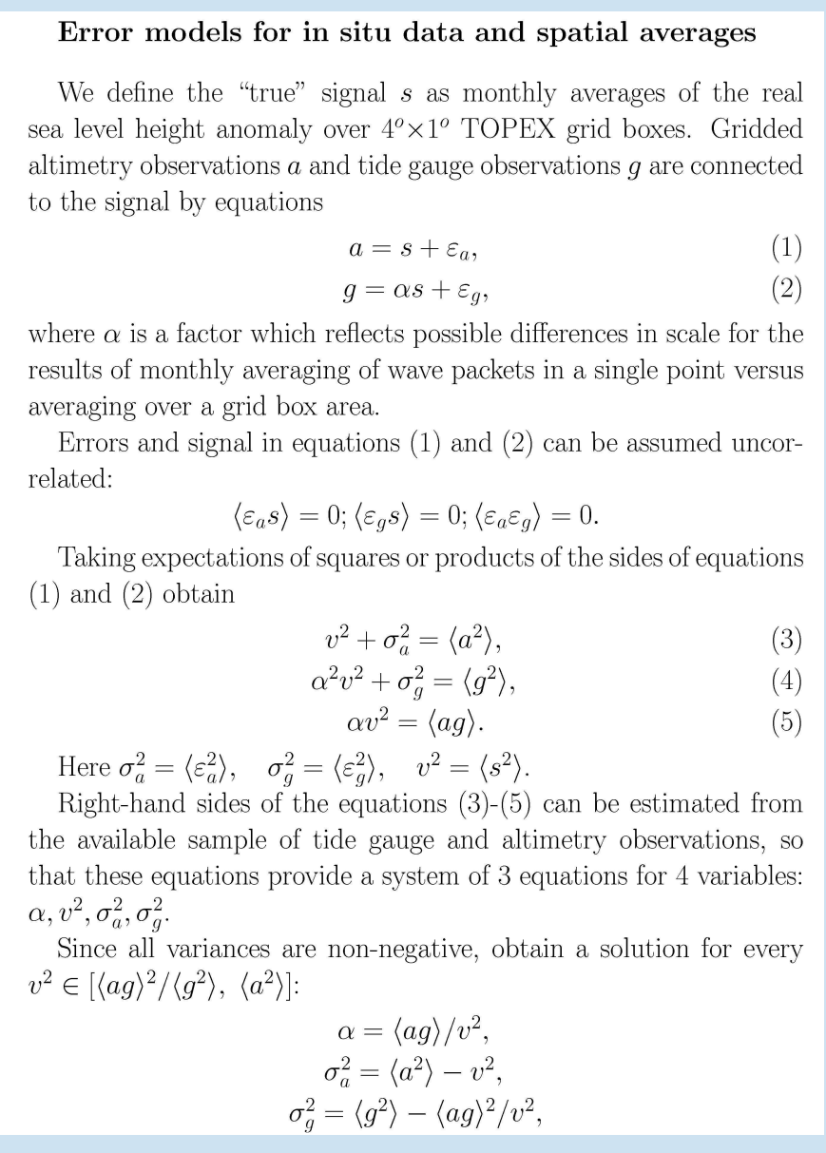
Validation of T/P error estimates by comparison with the tide gauge records, October 1992 -- March 2001. The top panel compares monthly tide gauge sea level height anomalies at Christmas Island (dashes) with altimetric measurements from the corresponding gridbox (centered at 2N and 158W) of *Cheney et al.* [1994] T/P product. Dots show values from individual altimetry passes, and the solid line shows their monthly averages for this gridbox. Temporal RMS values of the intrabox variability sigma inside the gridbox, the sampling error estimate r for the gridbox mean, and the RMS difference between the gridbox and tide gauge monthly means d are indicated as well. In the lower left panel, circles are differences between 31 tide gauges and T/P bins. Differences would fall along the solid line if the only errors were the ``optimistic" estimate of T/P errors. The dashed line inflates these optimistic estimates by a factor of 1.5. In the lower right panel, thin lines show constraints on the inflation factor alpha and tide gauge error r for individual tide gauges. The thick line shows the median constraint.

7.MODELING TIDE GAUGE ERROR


SSV ratio from tide gauges and TOPEX


Verification of tide gauge model error

8. SCALED TRANSFERS BETWEEN POINTWISE VALUES AND AREA AVERAGES



Error models for in situ data and spatial averages

We define the "true" signal s as monthly averages of the real sea level height anomaly over $4^o\times1^o$ TOPEX grid boxes. Gridded altimetry observations a and tide gauge observations g are connected to the signal by equations

$$a = s + \varepsilon_a, \tag{1}$$

$$g = \alpha s + \varepsilon_g, \tag{2}$$

where α is a factor which reflects possible differences in scale for the results of monthly averaging of wave packets in a single point versus averaging over a grid box area.

Errors and signal in equations (1) and (2) can be assumed uncorrelated:

$$\langle\varepsilon_a s\rangle = 0; \langle\varepsilon_g s\rangle = 0; \langle\varepsilon_a\varepsilon_g\rangle = 0.$$

Taking expectations of squares or products of the sides of equations (1) and (2) obtain

$$v^2 + \sigma_a^2 = \langle a^2\rangle, \tag{3}$$

$$\alpha^2 v^2 + \sigma_g^2 = \langle g^2\rangle, \tag{4}$$

$$\alpha v^2 = \langle ag\rangle. \tag{5}$$

Here $\sigma_a^2 = \langle\varepsilon_a^2\rangle, \quad \sigma_g^2 = \langle\varepsilon_g^2\rangle, \quad v^2 = \langle s^2\rangle$.

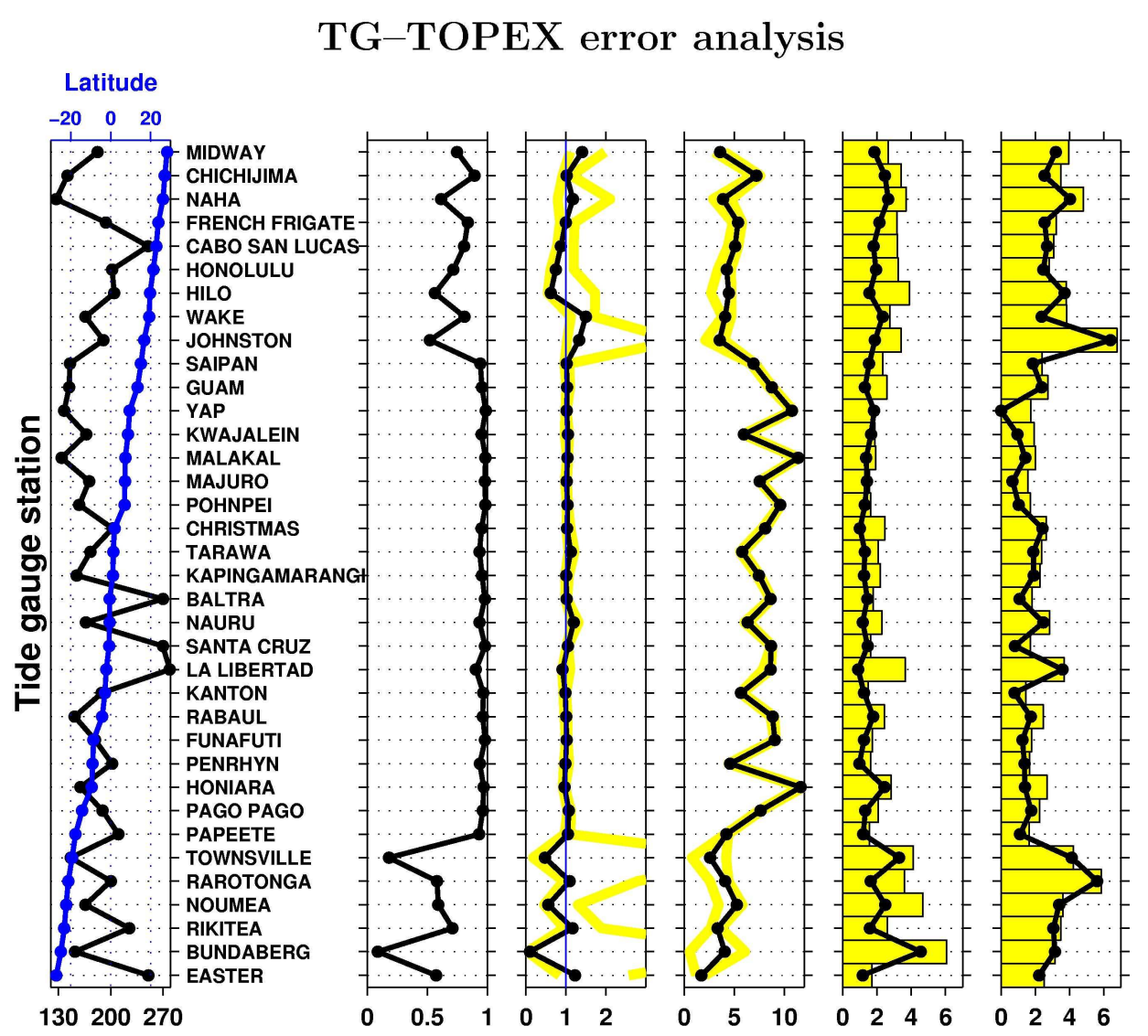
Right-hand sides of the equations (3)-(5) can be estimated from the available sample of tide gauge and altimetry observations, so that these equations provide a system of 3 equations for 4 variables: $\alpha, v^2, \sigma_a^2, \sigma_g^2$.

Since all variances are non-negative, obtain a solution for every $v^2 \in [\langle ag\rangle^2/\langle g^2\rangle,\ \langle a^2\rangle]$:

$$\alpha = \langle ag\rangle/v^2,$$

$$\sigma_a^2 = \langle a^2\rangle - v^2,$$

$$\sigma_g^2 = \langle g^2\rangle - \langle ag\rangle^2/v^2,$$



9. RESULTS

1.We have derived a connection between eddy kinetic energy and small-scale sea surface height variability; a coefficient in this connection characterizes the sea surface height wavenumber spectrum; its values for a high-resolution model are drastically different from those computed for altimetry fields.

2.We analyzed the ratios of temporal and spatial contributions to the small-scale sea surface height variability. This ratio has proved useful for modeling effective error in tide gauge data.

3. We introduced the formalism for "scaled" transfers between differently averaged values and applied it to the tropical Pacific tide gauges and altimetry data.

10. FURTHER WORK:

To use multisatellite altimetry data sets to combine approaches 1 and 2 in the "scaled" data error models 3 for assimilating in situ and satellite data.

11.REFERENCES:

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