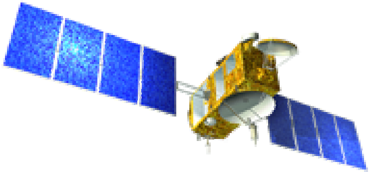
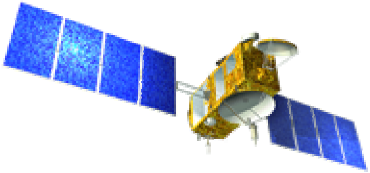


Assesment of the JASON-1 Look-Up Tables Using Multiple Gaussian Functions as an Approximation of the PTR

OZ.Zanifé(1), P. Thibaut(1), L. Amarouche(1), B. Picard(1), P. Vincent(2), N. Picot(3)

(1) : Collecte Localisation Satellites (CLS), Toulouse, FRANCE
(2) : IFREMER, Paris, FRANCE
(3) : Centre National d'Etudes Spatiales (CNES), Toulouse, FRANCE

- email : ozanife@cls.fr
- email : npicot@cnes.fr



INTRODUCTION

– The objective of this study is to compare the former JASON "operational retracking" (MLE3) which approximates the radar altimeter Point Target Response (PTR) with one gaussian with a retracking using several gaussians.

– Thus doing, this process validates the look up tables that are generated thanks to a complex simulator of the JASON radar altimeter since most of the effect comes from the approximation of the PTR.

– The following items are developed in the subsequents slides :

- Retracking with one gaussian function and with a sum of several gaussian functions
- approximation of the PTR by one and several gaussian functions
- impact on Brown/Hayne model
- comparison of retracking results
- conclusions

Retracking (1/2)

- The former JASON retracking is based on Maximum Likelihood Estimator and performs the extraction of 3 parameters : Epoch, Composite Sigma (related to the Significant Wave Height : SWH) and Amplitude through the following equations :

$$\theta_{m,n} = \theta_{m,n-1} - g\left(BB^T\right)^{-1}_{\theta_{m,n-1}} \left(BD\right)_{\theta_{m,n-1}}$$

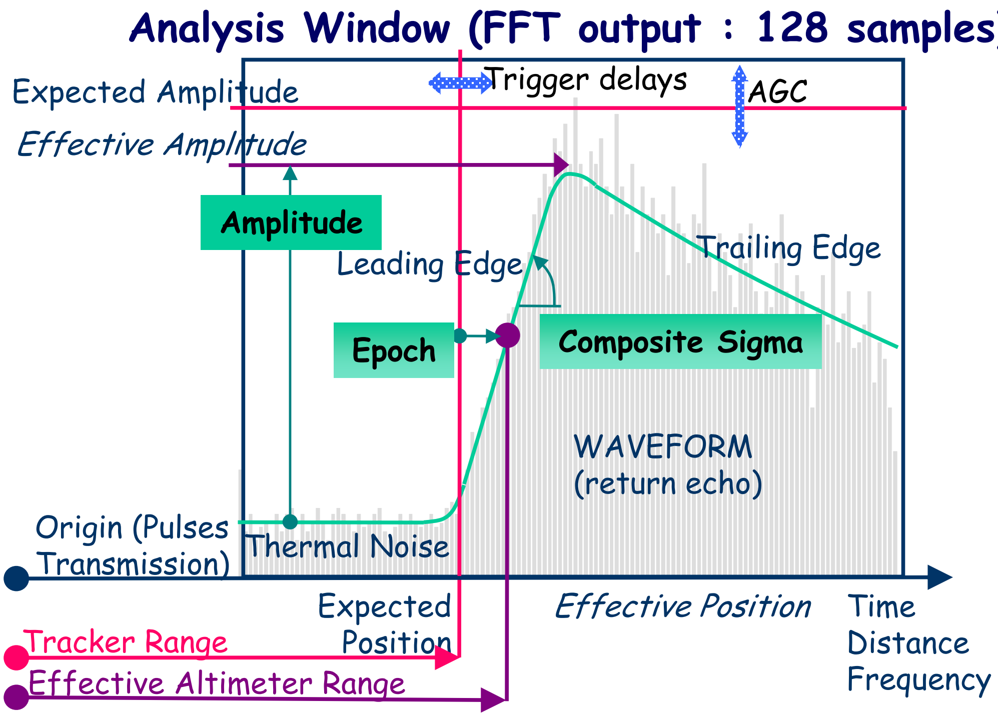
where $\theta_{m,n}$ are the parameters to be estimated.

m, n and g being respectively the index of the waveform sample, the current iteration and the gain.

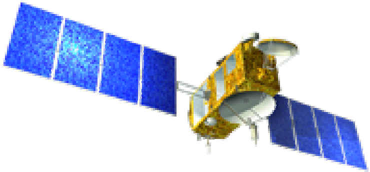
The derivative matrix B and the residual vector D being given by :

$$B_{m,k} = \frac{1}{Pu}\frac{\partial \overline{V_k}}{\partial \theta_m} \qquad D_{k,1} = \frac{\overline{V_k} - V_k}{Pu}$$

where V_k and $\overline{V}_k$ are respectively the measured and the modeled samples of the waveform (thanks to the Brown model).



Retracking (2/2)



- The Brown model (Hayne 1980) is used to analytically model the waveform. It is derived from the convolution of three terms :
• the flat sea surface response (FSR)
• the probability density function of the sea surface height (PDFh)
• the point target response of the radar altimeter (PTR)

- In the case of the JASON retracking, the PTR is aproximated by a single gaussian according to : σ_p = PTR width = 0.513*$\Delta\tau$ ($\Delta\tau$ = 3.125 ns) and lead to the following equation:

$$\overline{V}_k = \frac{P_u}{2}\left[1 + erf\left(\frac{t_k - \tau - \alpha\sigma_c^2}{\sqrt{2}\sigma_c}\right)\right]\exp\left[-\alpha\left(t_k - \tau - \frac{\alpha\sigma_c^2}{2}\right)\right] + P_n$$

and :

$$\sigma_c^2 = \sigma_p^2 + \sigma_s^2 \quad \text{related to the SWH} \quad SWH = 2c\sigma_s$$

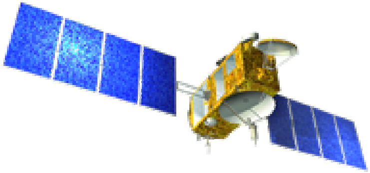
$$\alpha = \frac{4c}{\gamma h\left(1 + \frac{h}{R_e}\right)} \quad ; \quad \gamma = \frac{2}{Log_e(2)}.\sin^2(\frac{\theta_0}{2})$$

with :
τ : the epoch (related to range)
σ_c : the composite Sigma (related to SWH)
P_u : the amplitude (related to Sigma0)
P_n : the thermal noise

where :
c = velocity of light
h = mean satellite altitude
R_e = earth radius
θ_o = antenna beamwidth

- When using several gaussian functions to model the PTR, thanks to the convolution process, the new model is « simply» a sum of elementary Brown models using in each one the values of the individual PTR width. The same holds for the derivative matrix and the residual vector

Modeling of the PTR

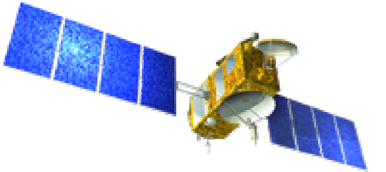


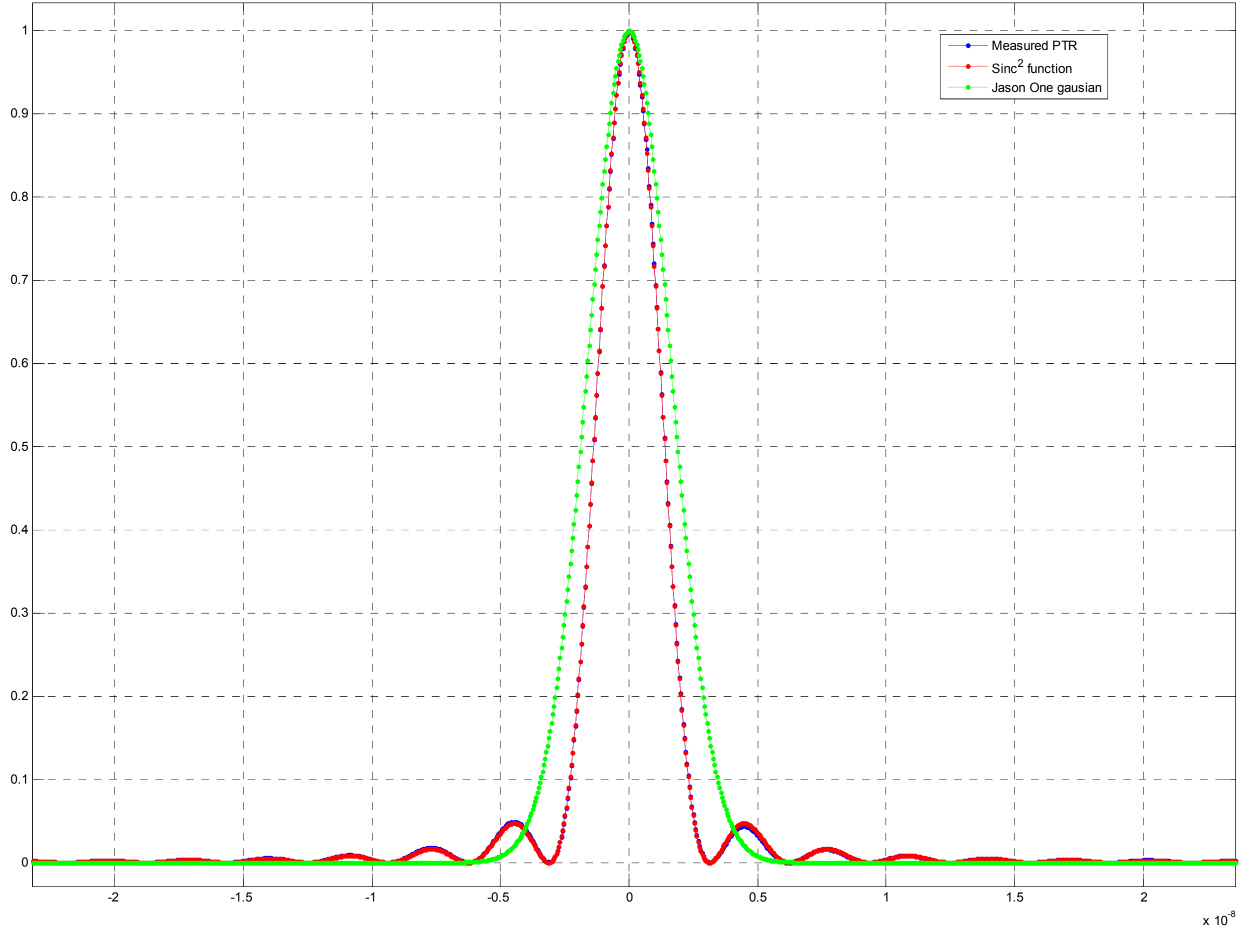
-The PTR is well approximated by a sinc function

- In the case of JASON as already mentioned it has been modeled by one gaussian function as indicated in the previous slide. It is important to notice that the "unmodeled" part of the PTR is taken into account trough the JASON look-up tables.

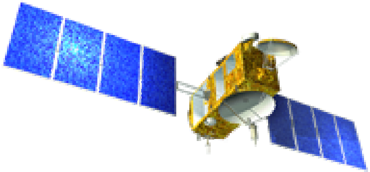
- In the frame of this study, up to 101 lobes (main lobe and 50 side lobes on each side of the main lobe) of the PTR have been modeled by gaussian functions. The modeling has been performed lobe by lobe thanks to a weighted least square method. For each lobe, the procedure is iterated until the residuals (PTR – sum of gaussian) are below a threshold (10-3 for the main lobe et 10-4 for the side lobes). Through this procedure, PTR was approximated by about the sum of about 120 gaussian functions.

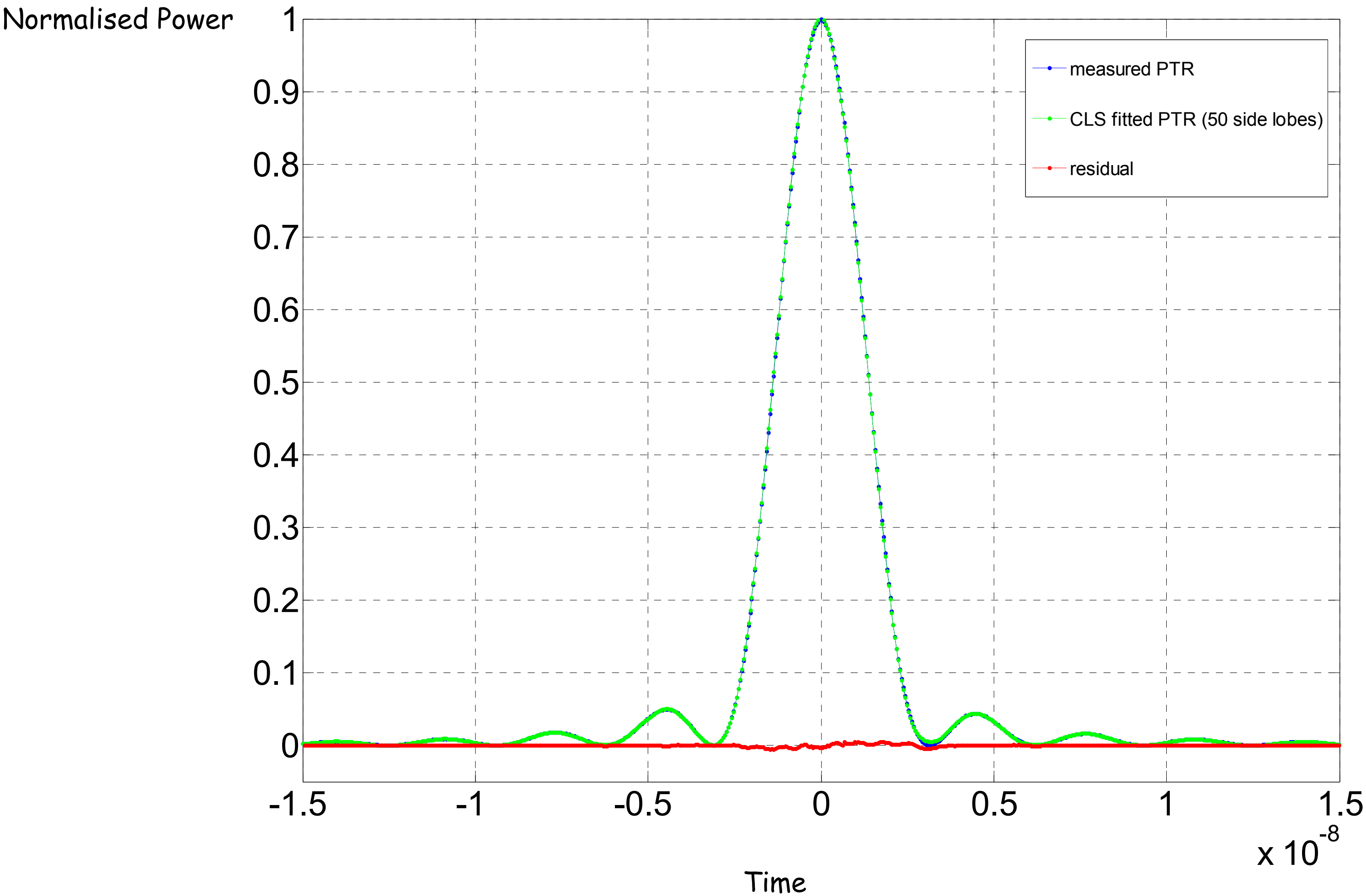
- In the subsequent slides comparisons between :
    - the measured PTR
    - the corresponding Sinc function
    - the Jason one gaussian approximation
    - the CLS fitting of the PTR (101 lobes)



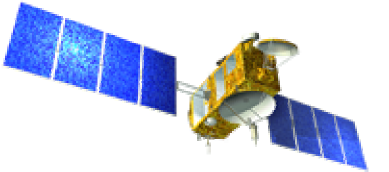


Measured PTR
$Sinc^2$ function
Jason One gausian
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x 10^{-8}




Normalised Power
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1.5
-1
-0.5
0
0.5
1
1.5
x 10^{-8}
Time
measured PTR
CLS fitted PTR (50 side lobes)
residual

Impact on Return Echo

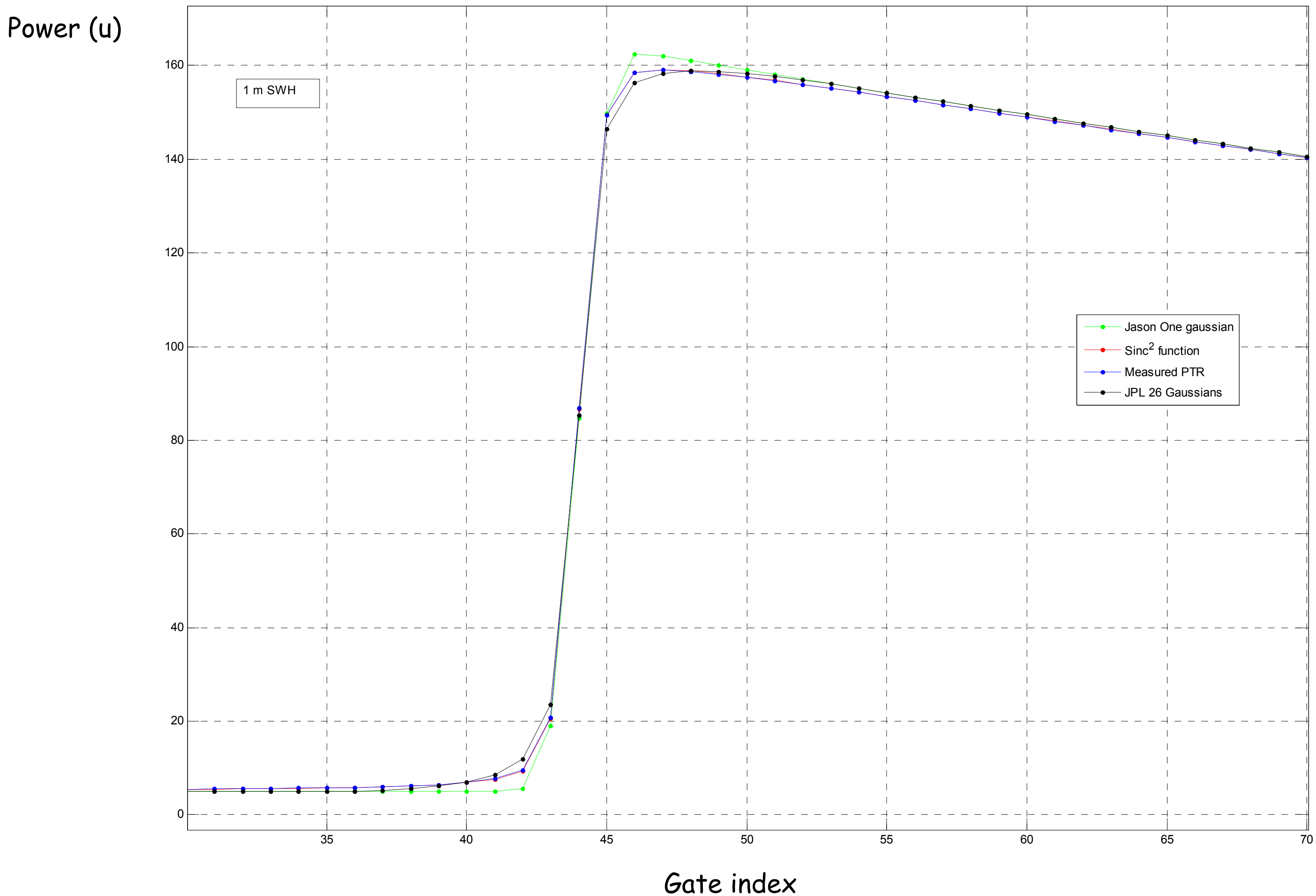


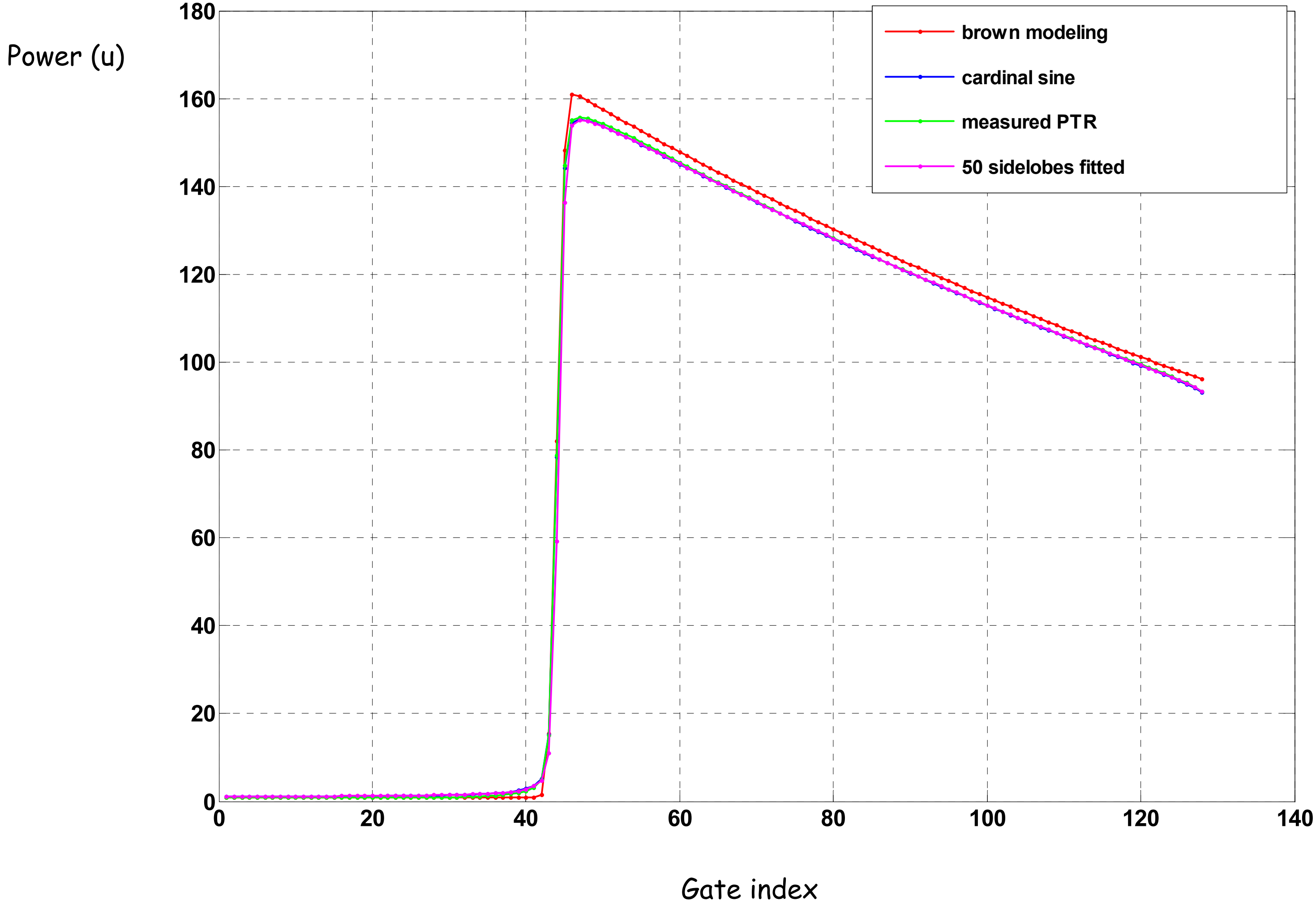
- The impact on the return echo is shown trough comparison between :
  - return echo obtained by performing the convolution using the measured PTR
  - return echo obtained by performing the convolution using the $sinc^2$ function
  - return echo obtained using the Brown model with one gaussian function (JASON retracking)
  - return echo obtained using the Brown model with the 120 gaussian functions

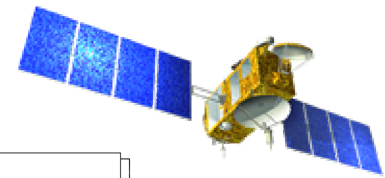
  for a SWH of 1 m.

- As one can notice, the return echoes are very close to each other except when using one single gaussian. The main differences appear at the begin and at the end of the leading, thus mainly affecting the determination of the SWH and related parameter such as the skewness parameter.

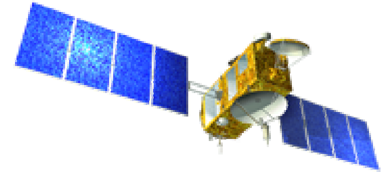
- The difference in amplitude in the plateau of the return echo is certainly due to a mis-scaling of the Brown model in our computation.


Power (u)
160
140
120
100
80
60
40
20
0
1 m SWH
Jason One gaussian
Sinc2 function
Measured PTR
JPL 26 Gaussians
35
40
45
50
55
60
65
70
Gate index


Power (u)
180
160
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
Gate index
brown modeling
cardinal sine
measured PTR
50 sidelobes fitted



Results



- The Ku band waveforms of 124 passes of JASON cycle 17 have been retracked with a ML3 (extraction of the 3 major parameters) using :
    - one single gaussian (JASON retracking)
    - CLS fitting procedure for 61, 81 and 101 lobes respectively named 30, 40, 50 side lobes

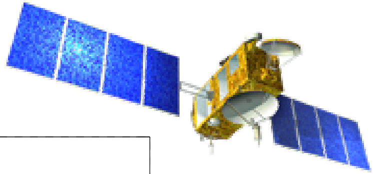
- First is given an histogram of the SWH for these 124 passes. If we take a threshold of 2 000, the results will be representative between 1 to 5 m SWH.

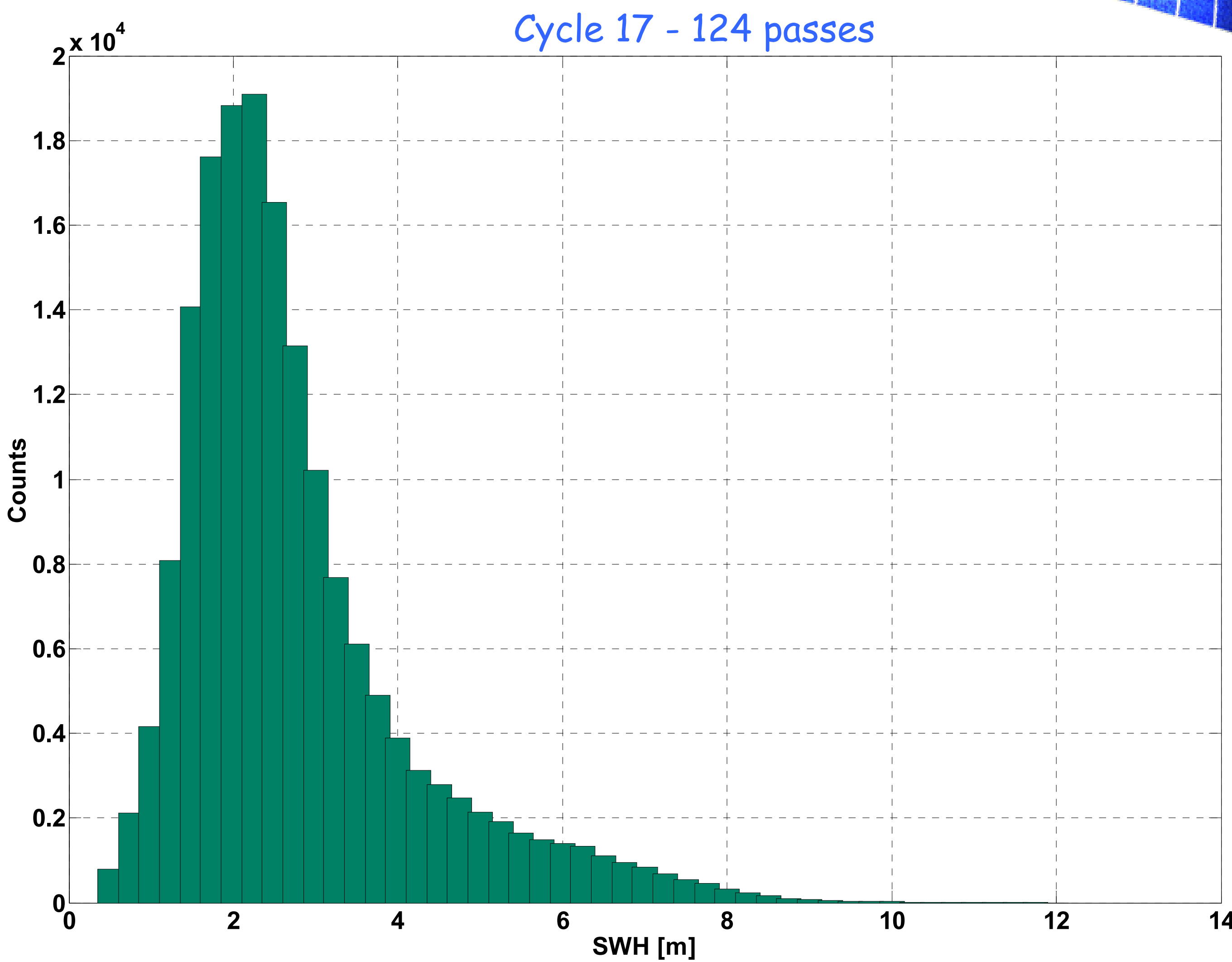
- The results are shown for SWH and the epoch. These are the differences (or residual) between the new extracted parameter and the corresponding parameter given in the product. Also given are the corresponding look-up tables for comparison.

- As one can notice, a very good agreement is found for SWH between the residual and the look-up table in the range of SWH validity within a level of 2 cm accuracy, except between 1 and 1.5 m. The more side lobes are integrated, better the results are.

- As far as the epoch (equivalent to range) is concerned, except for a bias between the two curve, a very good agreement is found to the level of a few millimeters.

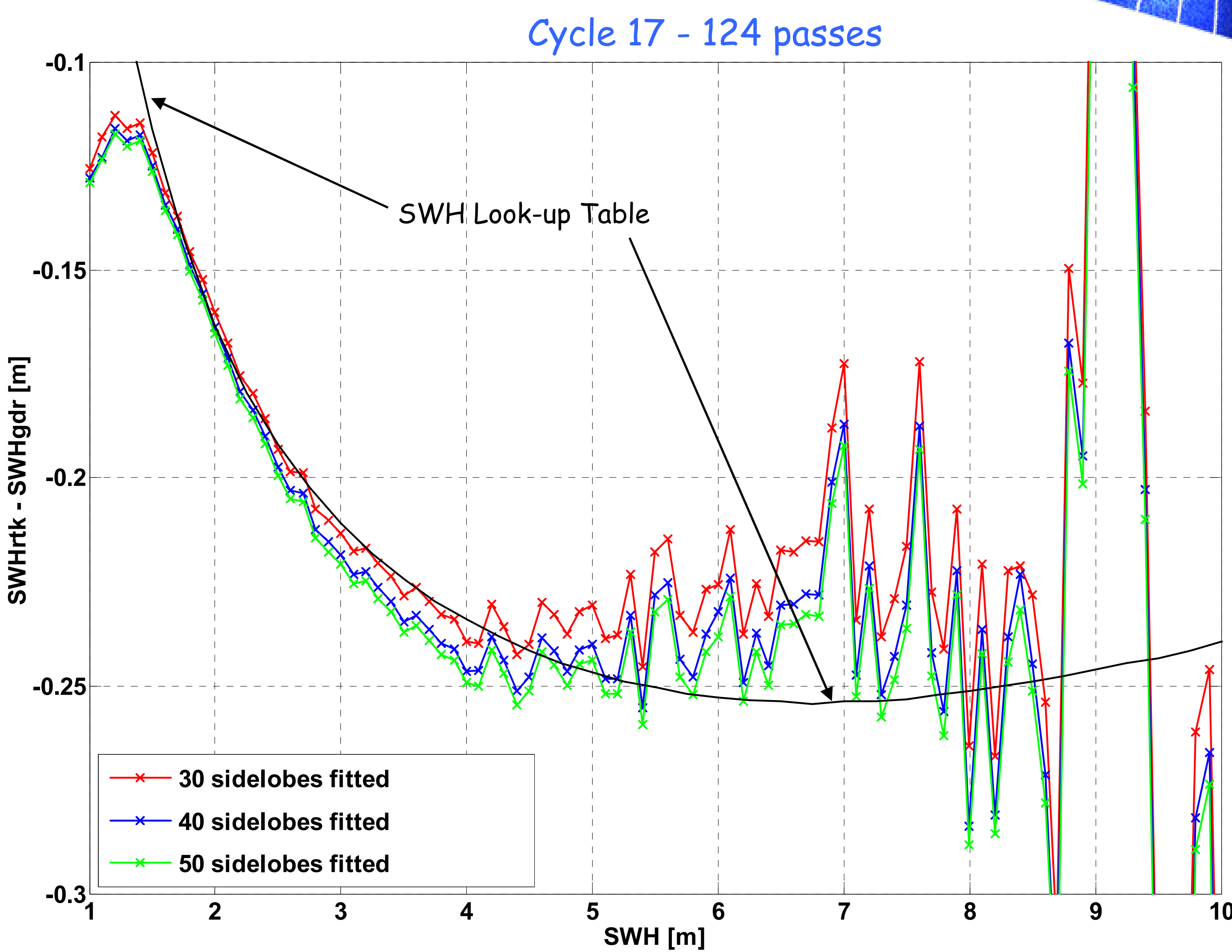
- These results are confirmed when using 2 cycles of retracked data (17 and 18)

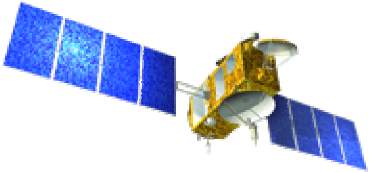


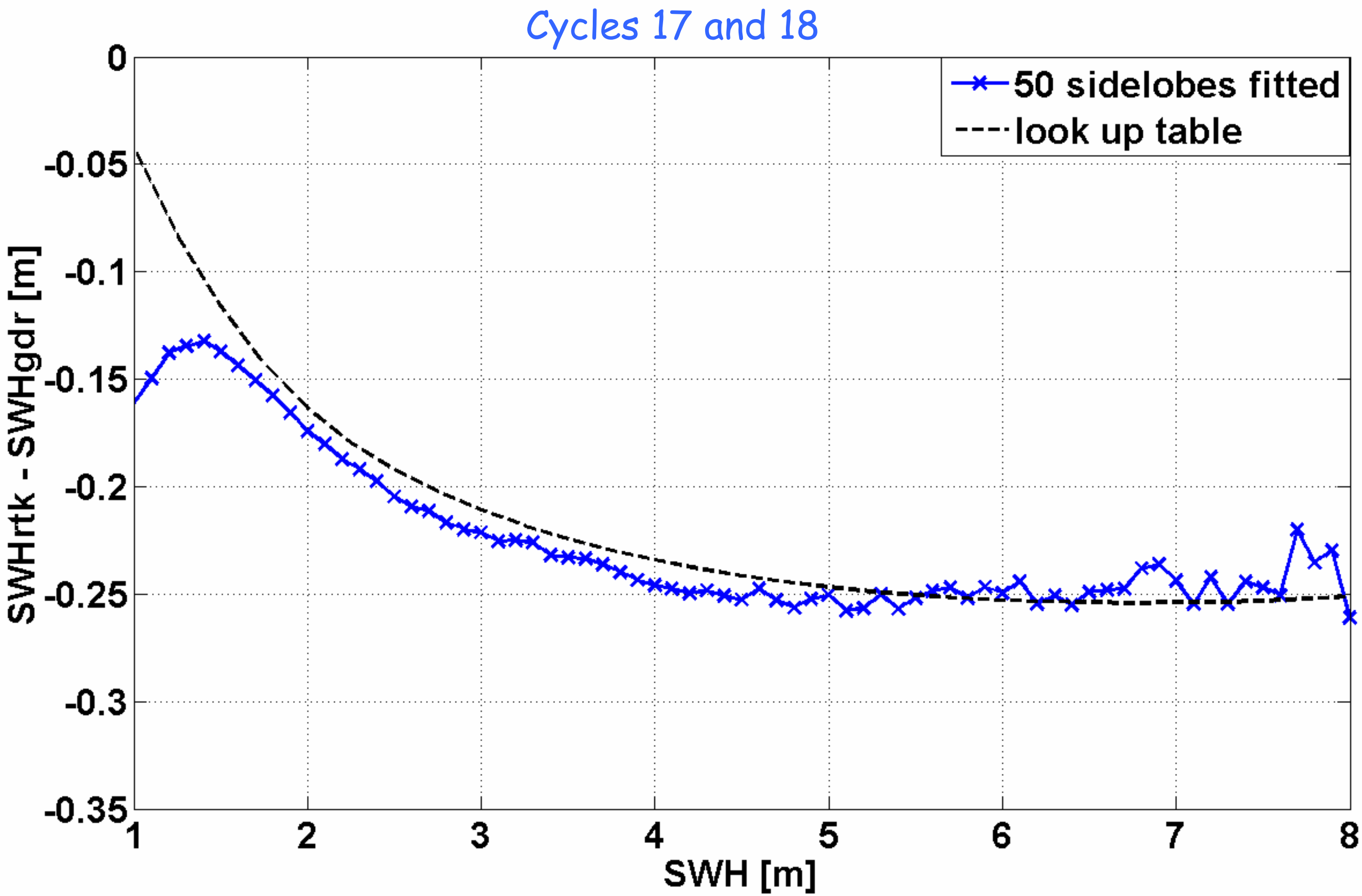


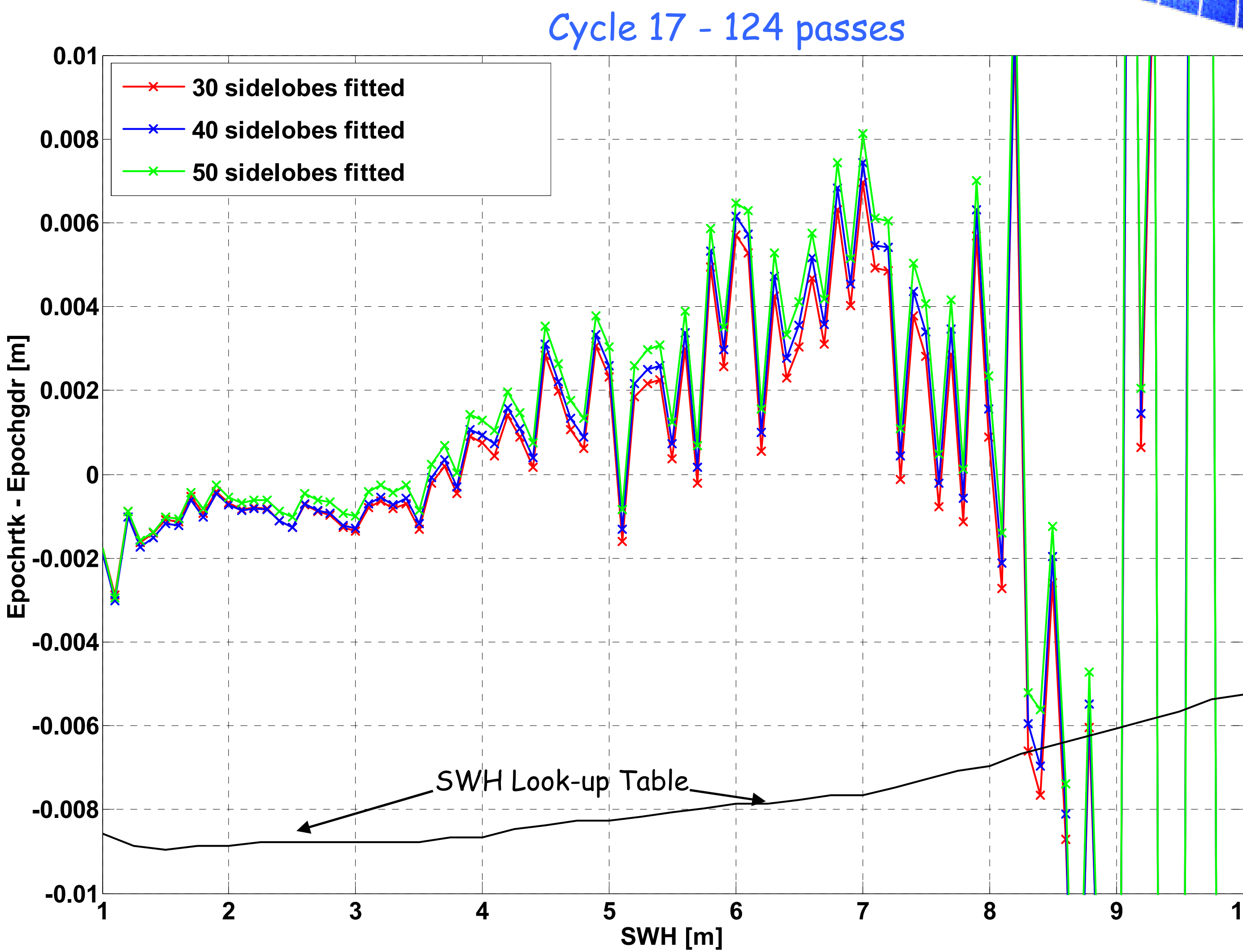
Cycle 17 - 124 passes
x 10^4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Counts
0
2
4
6
8
10
12
14
SWH [m]

OZ Zanifé & al

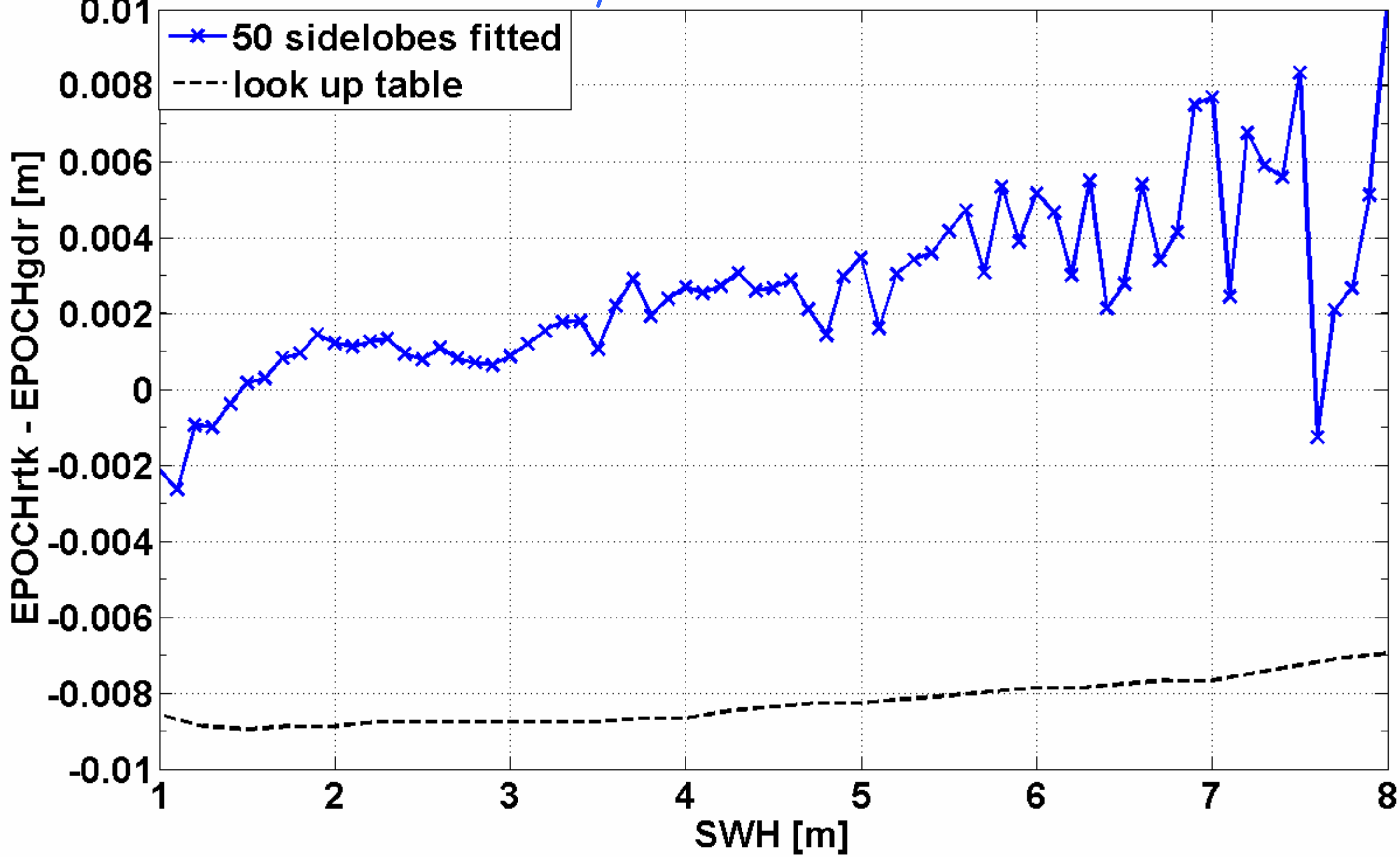

Cycle 17 - 124 passes
SWH Look-up Table
SWHrtk - SWHgdr [m]
SWH [m]
-0.1
-0.15
-0.2
-0.25
-0.3
1
2
3
4
5
6
7
8
9
10
30 sidelobes fitted
40 sidelobes fitted
50 sidelobes fitted



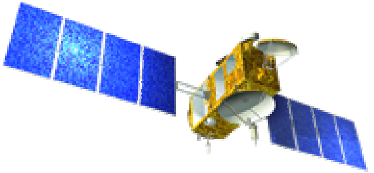

Cycles 17 and 18
SWHrtk - SWHgdr [m]
SWH [m]
50 sidelobes fitted
look up table
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
1
2
3
4
5
6
7
8


Cycle 17 - 124 passes
30 sidelobes fitted
40 sidelobes fitted
50 sidelobes fitted
Epochrtk - Epochgdr [m]
SWH [m]
SWH Look-up Table
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
1
2
3
4
5
6
7
8
9
10

Cycles 17 and 18



Conclusions



- JASON measured PTR is very close to theoretical Sinc^2 function

- JASON one gaussian approximation modelised main lobe and part of first side lobe. The effect of the remaining side lobes are taken into account by the look up tables

- When modeling more the first side lobe using gaussian, most of the impact is found on SWH, and has nearly no impact on range

- The modeling of the PTR by a sum of gaussian is able to recover the dynamic of the look-up tables and agreed with them to a level of :
    - 2 cm for SWH
    - 5 mm for the range

for SWH between 1.5 m and 6/7 m

- Further analysis is required for SWH below 1.5 m

- This exercise will be redone for the MLE4