Calculating the Ocean's Mean Dynamic Topography from a Mean Sea Surface and a Geoid

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Summary

The standard approach to calculating the ocean's mean dynamic topography (MDT) geodetically is to subtract a geoid model from a altimetric mean sea-surface (MSS) in a point-wise fashion. Here we present an alternative spectral method for computing the MDT that reduces the need for spatial filtering and the corresponding attenuation of ocean gradients. This is achieved by first expressing the MSS in spectral form allowing us to match omission errors in the MSS to those in the geoid.

1. The point-wise approach

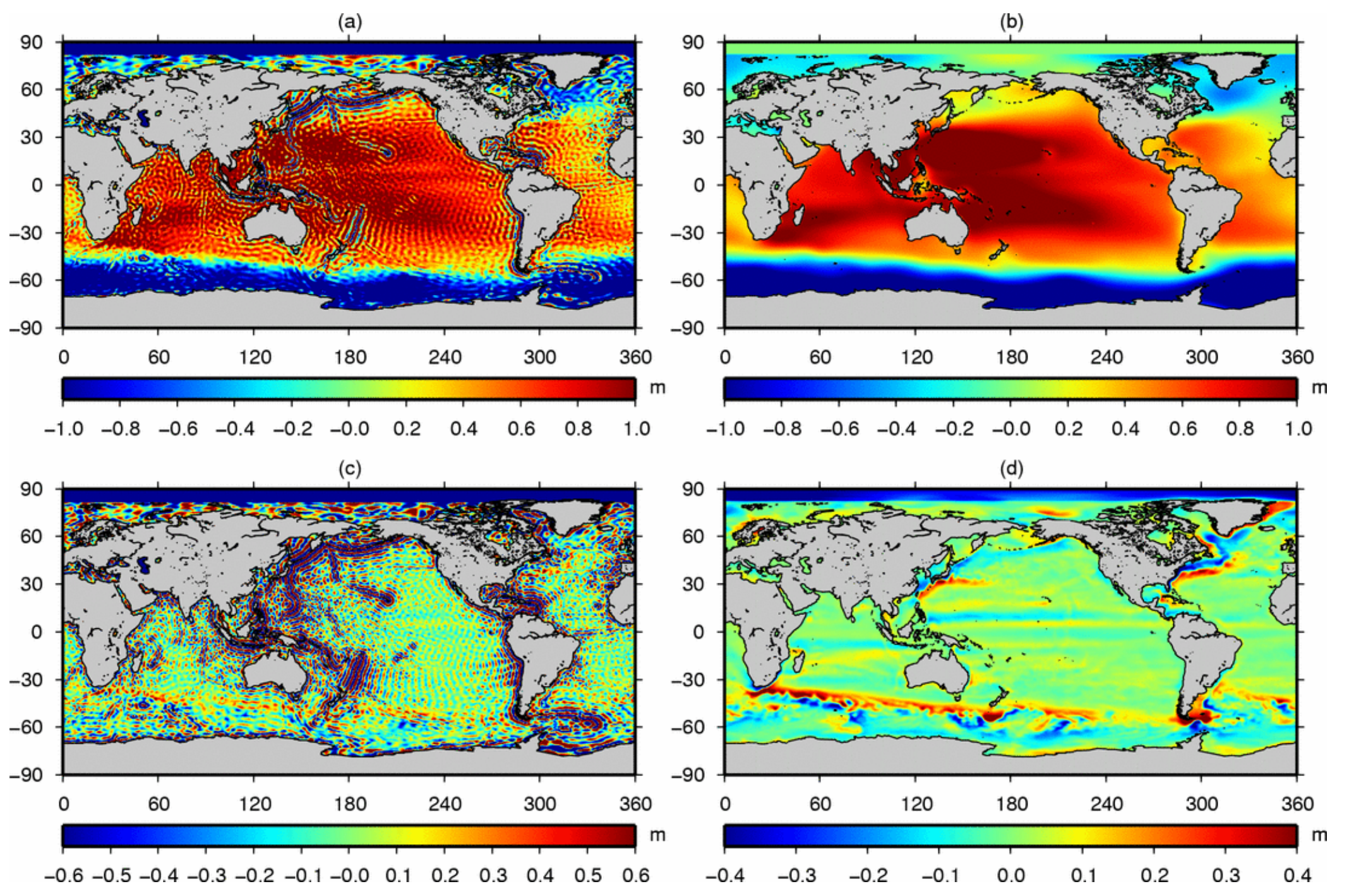

Figure 1. (a) An MDT computed in a point-wise fashion with the geoid model (GGM02S) truncated at degree 100. (b) The MDT smoothed with a Gaussian filter of 400km half-weight radius. (c) The filter residual obtained by subtracting the filtered from the unfiltered MDT. (d) A similar residual calculated from the OCCAM ocean model.

The simplest approach to calculating a geodetic MDT is to express the geoid height and the MSS on a common grid and subtract one from the other in a point-wise fashion.

However, this requires that we truncate the spectral model of the geoid resulting in the propagation of geoid omission errors into the MDT (figure a). Local omission errors result in small scales from the MSS being retained in the MDT. Non-local geoid omission errors in the MDT result from Gibbs fringes in the gridded geoid height field due to steep gradients in the gravity field.

While spatial average filtering can be used to remove this noise (figures b and c), this also results in the attenuation of the ocean signal (figures c and d). This leads us to look for an alternative approach that reduces the need for such filtering.

2. The spectral method (a)

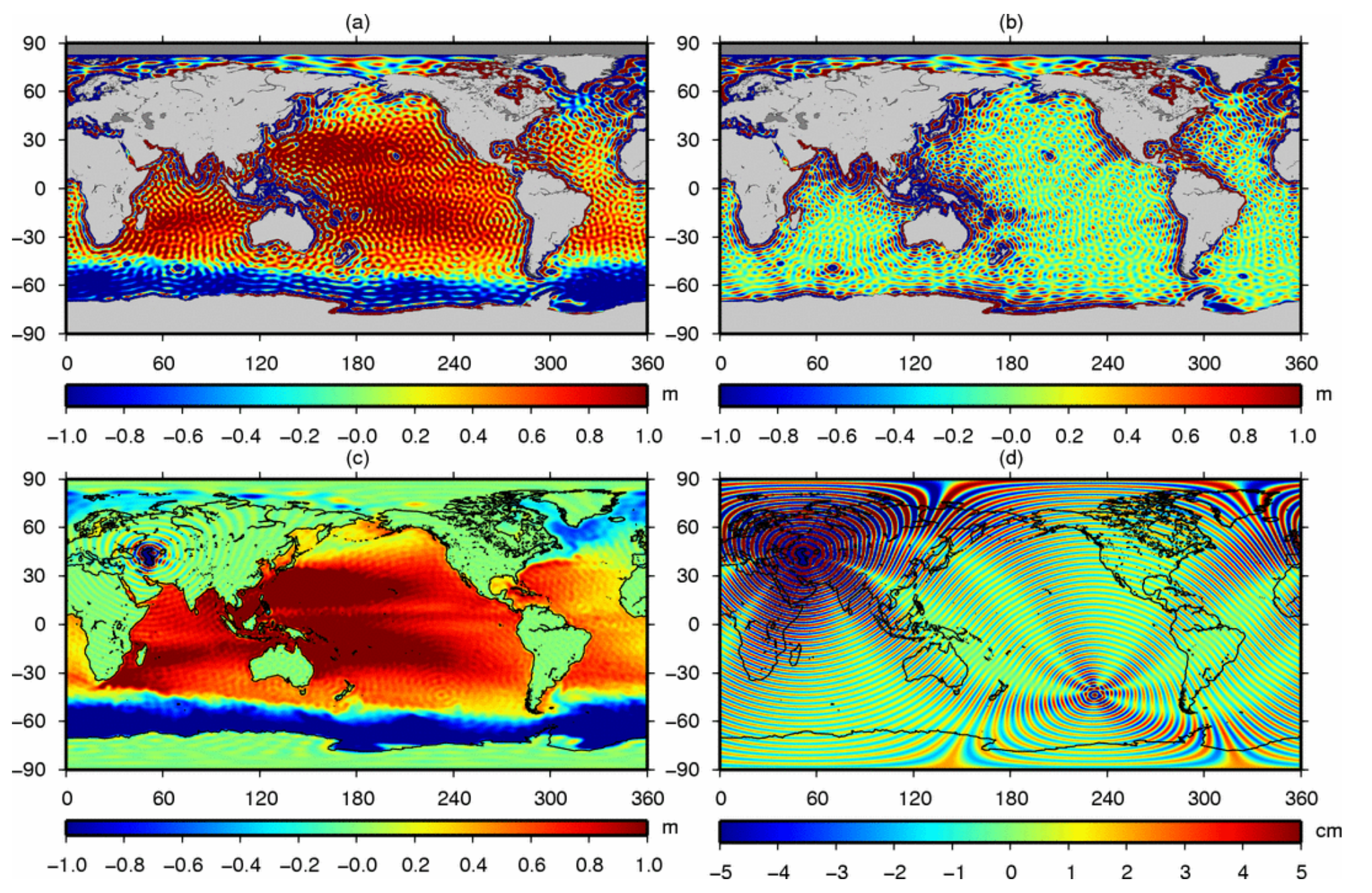

Figure 2. (a) An MDT computed by the spectral method with undefined MSS values set to zero. (b) The difference between the MDT shown in (a) and a spectral MDT with undefined MSS values set equal to the geoid model GGM02S as shown in (c). (d) The difference between the MDT shown in (d) and a similar MDT but with the Caspian Sea also replaced by GGM02S. All fields expanded do degree 100.

The spectral method for MDT computation expresses the MSS as a spectral model and expands only the MSS-geoid difference to obtain the MDT (see box for mathematical details). In this study we use the CLS01 MSS from Collecte Localisation Satellites and GRACE Gravity Model GGM02S from the Center for Space Research, Texas.

Since the MSS is not a global field a problem arises in what value to assign undefined regions. Using zero leads to severe Gibbs contamination of the resulting MDT (figures a and b) because of the discontinuity between defined and undefined regions. Given the close agreement between the MSS and geoid this problem is greatly reduced by forming a MSS/geoid hybrid surface, with undefined regions of the MSS set equal to the geoid height (figure c).

Interestingly, because it is well below the geoid, retaining the Caspian Sea MSS in this hybrid surface, results in Gibbs effects that contaminate the MDT globally (figures c and d). These are removed by replacing the MSS with the geoid in this region.

Spectral method mathematics:

It can be shown that the spectral model of the equivalent disturbing potential of the geoid plus the MDT (i.e. the MSS) is given by:

$$\begin{Bmatrix} C^H_{lm} \\ S^H_{lm} \end{Bmatrix} = \frac{r_e^{l+1}}{4\pi GMa^l}\int_S \gamma(\theta')H(\theta,\phi)\begin{Bmatrix} \cos(m\phi) \\ \sin(m\phi) \end{Bmatrix}\widetilde{P}_{lm}(\cos\theta)ds$$

Then the MDT can be computed in the spectral domain using:

$$\{C,S\}^\eta_{lm} = \{C,S\}^H_{lm} - \{C,S\}^N_{lm}$$

Finally the MDT can be expressed spatially using:

$$\eta(\theta,\phi) = \frac{GM}{r_e\gamma(\theta')}\sum_{l=0}^{L}\left(\frac{a}{r_e}\right)^l\sum_{m=0}^{l}\widetilde{P}_{lm}(\cos\theta)\{C^\eta_{lm}\cos(m\phi)+S^\eta_{lm}\sin(m\phi)\}$$

where C_{lm} and S_{lm} are spherical harmonic coefficients of degree l and order m, H is the MSS height, N is the geoid height, η is the MDT, P_{lm} are the associated Legendre functions, θ is geocentric colatitude, Φ is longitude, GM is earth's gravitational mass constant, a is the earth's equatorial radius, $r_e=r_e(\theta)$ is the radius of the reference ellipsoid, θ' is geodetic colatitude, and γ is normal gravity on the surface of the reference ellipsoid.

3. The spectral method (b)

Note that although the CLS01 MSS used in this study is supplied as a MSS/EGM96 hybrid surface the difference over land between the EGM96 and the geoid (GGM02S) used for the MDT computation (figure a), the Gibbs fringes in the truncated hybrid surface do not match those in GGM02S leading to spurious features in the MDT (figure b).

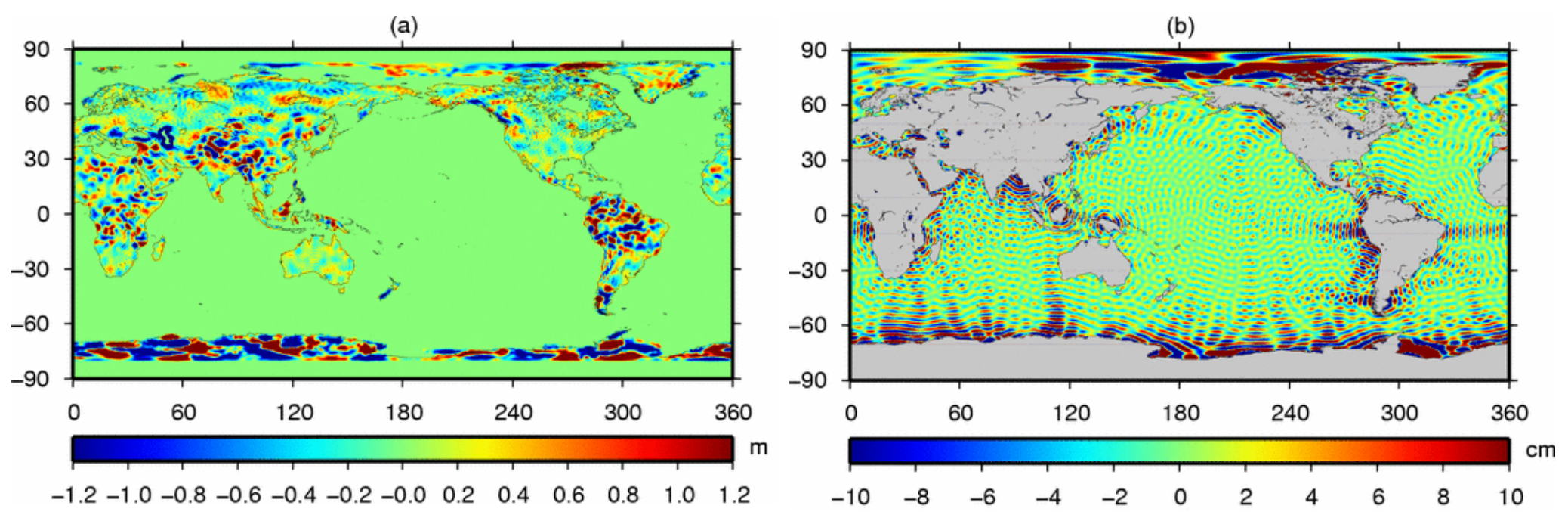

Figure 3. (a) The difference between EGM96 and GGM02S over undefined MSS regions (b) The difference between a spectral MDT computed to degree 100 using the CLS01 MSS as supplied with EGM96 in undefined regions and a spectral MDT with GGM02S replacing EGM96.

4. The spectral MDT

The spectral method produces a smoother MDT than the point-wise method without the need for undesirable spatial average filtering (figures a and b).

This is achieved by truncating the spectral model of the MSS/geoid hybrid to the same degree as the geoid so that over the ocean the omission errors in the MSS match those in the geoid. Alternatively, we are now truncating an MDT that does not have the the steep gradients of the MSS or geoid and a much smaller step to zero at sea/land boundaries.

Note there are two processes at work here: (i) a smoothing process whereby small scale features are removed from the MSS; (ii) conversely spurious Gibbs effects are introduced to the MSS to negate those in the truncated geoid.

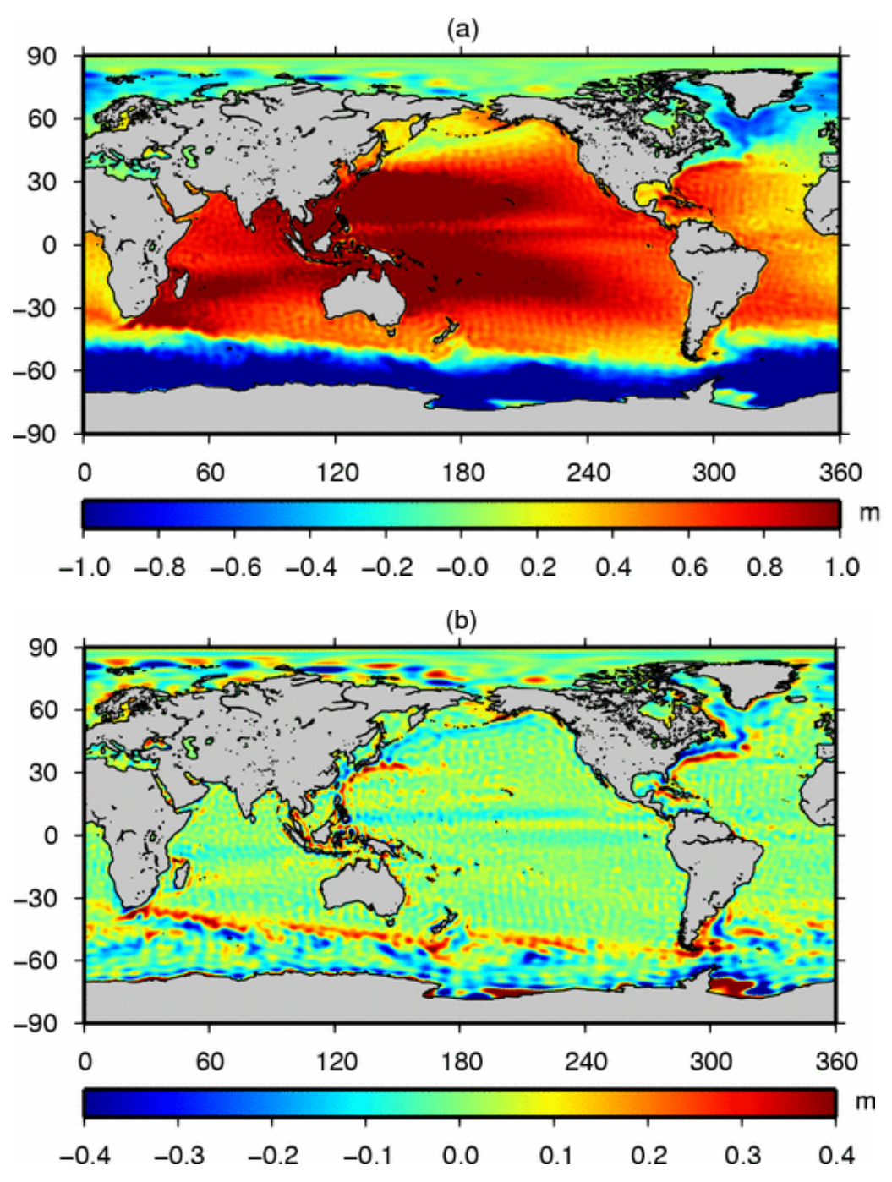

Figure 4. (a) A spectral MDT computed to degree 100 with all undefined regions and the Caspian Sea in the CLS01 MSS replaced with GGM02S values. (b) The filter residual obtained by subtracting from the unfiltered MDT an MDT filtered with a Gaussian filter of 400km half-weight radius.

5. Inter-comparison

Of course, since the spectral method includes smoothing (although as discussed above spatial smoothing is not the only reason why the spectral MDT is less noisy) for a fair comparison of the point-wise and spectral methods we must first filter the point-wise MDT. In fact, there is also noise in the spectral MDT so it too must be smoothed.

We found that a 200km Gaussian filter is sufficient to remove the residual noise in the spectral MDT (figure a), where as with this filter much noise remains in the point-wise MDT (figures b and c). In fact to obtain a comparably smooth point-wise requires a filter radius of 300km and even then problems remain at coastlines due to geoid omission error not easily removed by spatial averaging.

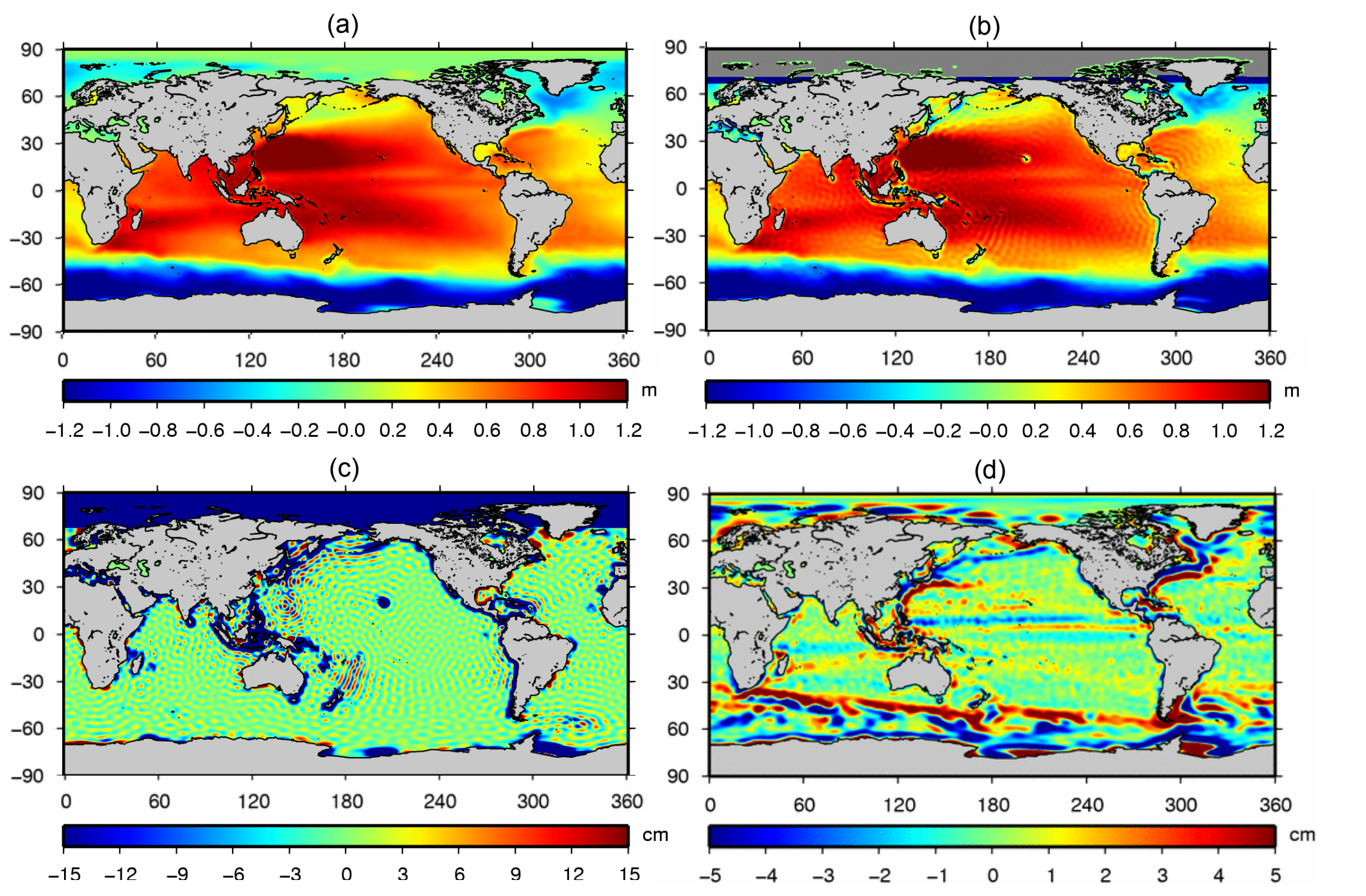

Figure 5. (a) The spectral MDT expanded to degree 100 and smoothed with a Gaussian filter of 200km half weight radius. (b) The point-wise MDT computed with GGM02S expanded to degree 100 and smoothed with a Gaussian filter of 200km half weight radius. (c) The difference between the smoothed MDTs show in (a) an (b). (d) The difference between the spectral MDT shown in (a) and the same MDT but smoothed with a Gaussian filter of half-weight radius 300km.

The key point is that more severe smoothing is required to remove noise from point-wise MDT and therefore more oceanographic information lost (figure d).

6. Conclusions

By matching omission errors in the MSS and geoid the spectral method for computing the MDT reduces the need for spatial smoothing. Since such smoothing attenuates MDT gradients the spectral method allows the better determination of the geostrophic surface currents. Care is needed however, in how the MSS is expressed as a spectral model. Further work is on-going in how to reduce problems associated with the discontinuity where the MSS meets the geoid.

Acknowledgements

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