Control of a free-surface barotropic model of the Bay of Biscay by assimilation of sea-level data in presence of atmospheric forcing errors

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Abstract

The purpose of this study is to assimilate various altimetry and tide-gauges data in the **barotropic, free-surface, finite element model MOG2D**, covering the **Bay of Biscay** and nested in a **North East Atlantic domain**. In a first step, we **explore the errors sub-space** of the model in presence of **forcing uncertainties**, and especially in presence of **high frequency atmospheric forcing errors**. This is done by an **ensemble modelling approach** (Monte-Carlo) in which the atmospheric fields are perturbed in a **multivariate way** : by generating an *a priori* ensemble of perturbed atmospheric forcing fields (10 meters wind and surface pressure from **ARPEGE meteorological model**), and computing the corresponding *a posteriori* ensemble of model states, one can **approximate the forecast errors** of the model by ensemble spread statistics. These statistics are shown to be neither homogeneous over the domain, nor stationary, since they are very dependent on the meteorological forcing. Then, the forecast covariance matrix is modelized through **forecast error Ensemble EOFs**. These statistics, in form of 6D-EOFs (Sea Level Anomaly, barotropic velocities, surface pressure and wind-stress components), are used in a **reduced-order sequential scheme**, **SEQUOIA**, used in an **Optimal Interpolation configuration** with the **MANTA** kernel developed at LEGOS/POC (De Mey, 2005), to constrain the model forecast in the framework of **twin experiments**. In a reference experiment, the data assimilation system is calibrated and sensitivity tests are conducted. The system provides significant error reduction for all state vector variables, but appears to be sensitive to configuration parameters : particularly, one need to **constrain atmospheric forcing fields** to achieve an efficient control of the model errors. Finally, the capability of realistic observing networks to reduce the model errors are compared. **Frequent and regularly spaced observations**, such as tide-gauges (SLA) or HF radars and buoys (velocity), appeared to be more adapted to the present data assimilation configuration than altimetry data.

1 - Model configuration

MOG2D model (Lynch and Gray (1979), adapted by Greenberg and Lyard)

- barotropic
- Non linear
- Finite Element method for spatial resolution (refine study in coastal and steeper bathymetry area)
- zone = Bay of Biscay (BoB) + English Channel (EC) + Celtic Sea (CS), nested in European shelf area (**Fig. 1**)
- Sea Level Anomaly, barotropic velocities

Configuration :

- Atmospheric forcing : surface pressure and 10 meters-wind velocity fields derived from atmospheric models ARPEGE products.
- Tide forcing
- European shelf solution used as open boundary conditions
- Time period : 16/11/1999, 00:00➔01/12/1999, 00:00

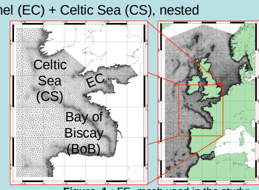


Figure. 1 : FE mesh used in the study: Bay of Biscay (BoB) + English Channel (EC) + Celtic Sea (CS) nested in European shelf

2 - Barotropic dynamics in response to atmospheric forcing

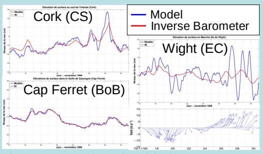


Figure. 2 : comparison of model and IB SLA response to atmospheric forcing in 3 points of the area

Different dynamics behaviour in the various seas (Fig. 2) :

- BoB : controlled by pressure temporal scales (ts) ~ 2 days
- EC : non-isostatic response ts ~ 1 day
- CS : intermediate response

The non-isostatic dynamic is predominant in EC, with Kelvin wave propagation (along both French and English coasts) and stationary processes; it's much weaker elsewhere.

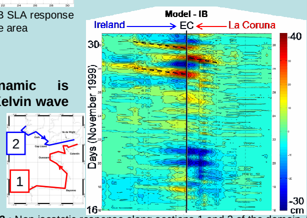
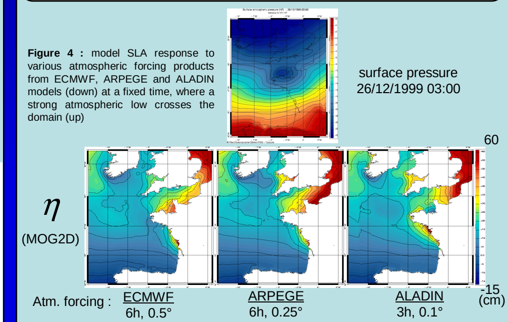


Figure. 3 : Non-isostatic response along sections 1 and 2 of the domain

3 - Impact of atmospheric model differences on oceanic model results

Figure 4 : model SLA response to various atmospheric forcing products from ECMWF, ARPEGE and ALADIN models (down) at a fixed time, where a strong atmospheric low crosses the domain (up)



➔ **Significant differences :** storm surge structures on French Atlantic coasts, differences patches in EC

4 - Ensemble modelling approach

As a prior requirement (and a research subject) for data assimilation, the specification of model errors has shown to be much more complicated in Shelf and Coastal Seas (hereafter SCS) than in the open ocean : SCS model errors appear to be inhomogeneous, non-stationary, anisotropic and multi-scale (Echevin *et al.*, 2000; Auclair *et al.*, 2003; Mourre *et al.*, 2004), due to strong non-linearity of SCS dynamic processes, intense control of coastlines and bathymetry, and fast response to atmospheric forcing. In our study, the forecast errors are approximated from ***Ensemble (Monte Carlo) simulations*** of the model in **response to atmospheric forcing** (p,τ) **errors**.

Perturbation strategy and ensemble simulation:

Figure 5 outlines the perturbation strategy/ensemble modelling approach we implemented in the study :

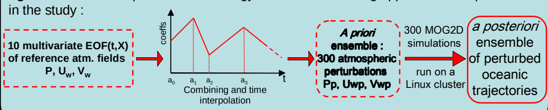


Figure 5 : perturbation strategy/ensemble modelling approach

5 - Characterization of model errors via ensemble statistics

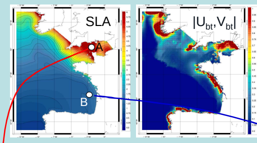


Figure 6 : time average of ensemble variance for SLA and barotropic current components

➔ **Inhomogeneous distribution of oceanic errors (Fig.6) :**

- SLA : max. error structures in EC, weaker in BoB
- U_{bt}, V_{bt} : mainly in the shallow coastal band and around cape zone, negligible in BoB

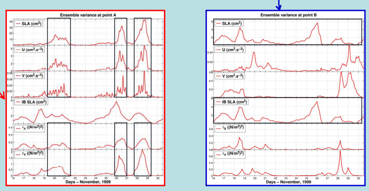


Figure 7 : time evolution of ensemble variance for SLA, U_{bt}, V_{bt}, IB SLA, τ_X , τ_Y at points A and B

➔ **Oceanic errors are non-stationary (Fig.7)**

- ~24h error growth
- closely following atmospheric error development :
  - In EC (A) : oceanic errors are mainly wind-driven
  - In BoB (B) : SLA errors controlled by pressure errors; velocity errors partially correlated to wind-stress uncertainties

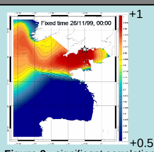


Figure 8 : significant correlation of a SLA measure at Weymouth with SLA field

Domain of influence (doi) of an isolated SLA observation at fixed time (Fig.8) :

- Characteristic dimension ~ atmospheric synoptic scales (~1000 km)
- Anisotropic structure
- High time variability

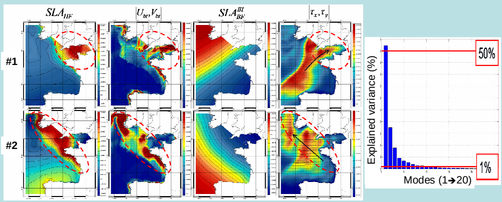


Figure 9 : oceanic error multivariate EOFs (first two dominant modes) and spectrum of variance explained by each mode (the first 20)

Covariant error structures in form of oceanic error multivariate EOFs (Fig.9)

100 EOFs were calculated using ensemble members as samples at 5 various dates in order to take non-stationarity of errors into account

- Red spectrum ➔ good representativity of error structures
- 1st mode : error "regime" in EC
- 2nd mode : shelf error "regime"

6 - Data assimilation methodology

We implemented the sequential **reduced-order** data assimilation code **SEQUOIA**, used in an **Optimal Interpolation configuration** with the **MANTA** kernel (De Mey, 2005).

Analysis step : $\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f\right)$

Order reduction

$$\mathbf{P}^f \approx \mathbf{S}^T \mathbf{P}_r^f \mathbf{S}$$

- $\mathbf{P}^f$: forecast error covariances matrix
- $\mathbf{S}$: **error EOFs approximated by ensemble EOFs (*cf.* §5)**
- $\mathbf{P}_r^f = diag(\nu_1,..,\nu_r)$ ν_k : error variances of mode *k*

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{S}^T\mathbf{K}_r\left(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f\right)$$

$$\mathbf{K}_r = \left(\mathbf{P}_r^f\right)^{1/2}\left(I + \rho_r\mathbf{R}_r^{-1}\rho_r^T\right)^{-1}\rho_r\mathbf{R}_r^{-1}$$

(Pham *et al.*, 1998)

- $\mathbf{R}$: observation error covariances matrix
- $\rho_r = \left(\mathbf{P}_r^f\right)^{1/2}\mathbf{H}_r^T$: reduced-order (RO) representer
- $\mathbf{H}_r = \mathbf{H}\mathbf{S}^T$: RO observation operator

choice of the state vector :

- Oceanic error strongly correlated to atm. errors
- Fast evolution of atm. conditions

➔ Need to correct the atmospheric forcing in the analysis step ➔ State vector : $SLA_{HF}\ U_{bt}\ V_{bt}\ SLA^{BI}_{BF}\ \tau_X\ \tau_Y$

7 - Control of model errors via data assimilation

The control of model errors due to atmospheric forcing uncertainties is achieved in the **framework of twin experiment** : a particular member of the perturbed oceanic trajectories ensemble is taken as a **"control" simulation**, from which **simulated observations** are extracted (and randomly noise added) at **10 tide-gauges locations** (see **Fig.10**), and then assimilated in another member of the ensemble, a so-called **"free" simulation**.

Reference configuration / obs. network

- 10 real tide gauges (10TG)
- 1.5 cm rms error
- Obs : $\Delta t = 1h$
- $\Delta\mathbf{T}^{assim} = 12h$
- Mean EOFs (*cf.* §5)

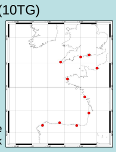


Figure 10 : 10 TG reference network

Results

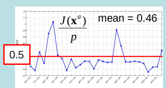


Figure 11 : J_{min} diagnostic

➔ mean(J_{min})~0.5 ensured for ensemble variance=1/3 V_i

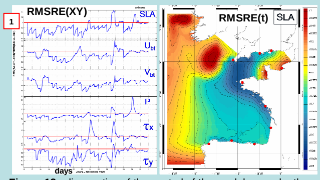


Figure 12 : diagnostics of the control of the model errors by the system

- Significant error reduction for all variables
- visible degradation at the end of the period (storm) : weak representativity of error EOFs
- correction zone located around TG

Diagnostics

- cost function minimum $J_{\min}$: qualify the intern coherency of the system (Talagrand, 1999, ECMWF)

$$E\left[J\left(\mathbf{x}^a\right)\right] = \frac{p}{2}$$ with *p* the number of assimilated obs.

- RMS Error Reduction :

RMSER(XY) : $$\frac{rms^{x,y}\left(x^{assim} - x^{ctrl}\right)}{rms^{x,y}\left(x^{libre} - x^{ctrl}\right)}(t) \overset{?}{\leq} 1$$

RMSER(t) : $$\frac{rms^{t}\left(x^{assim} - x^{ctrl}\right)}{rms^{t}\left(x^{libre} - x^{ctrl}\right)}(x,y) \overset{?}{\leq} 1$$

Sensitivity tests

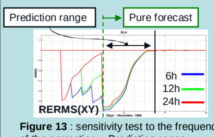


Figure 13 : sensitivity test to the frequency of the correction – Prediction range

- Best control for 6h-analysis
- Prediction range (PR) ~ 30h
- Small impact of correction frequency on PR

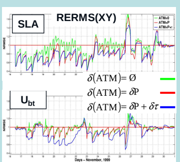


Figure 14 : sensitivity test to atmospheric correction

Need to correct the atmospheric forcing in order to efficiently control oceanic errors

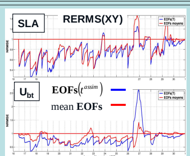
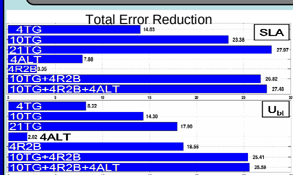


Figure 15 : sensitivity test to time-dependency of error statistics

Use of pre-computed EOFs at analysis time :

- best correction
- faster error growth

8 - Observation network performances



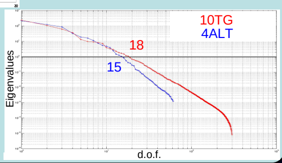
4TG, 10TG, 21TG : 4,10,21 tide-gauges
4ALT : Jason+Topex+GFO+Envisat – 15 days
4R2B : 4 HF radars sites + 2 anchored buoys

- need for frequent and regularly spaced observations
- methodological approach maladapted to altimetry network
- complementarity of SLA and velocity data
- relevance of a TG + buoy + HF radars

Eigenvalues spectrum of the representer matrix (scaled by R) :

Measure how many degrees of freedom of the system errors are captured by the obs. network (independently of DA) $\mathbf{R}^{-1/2}\ \mathbf{H}\ \mathbf{P}^f\mathbf{H}^T\ \mathbf{R}^{-1/2}$

Similar performance levels over dominant (large-scale) d.o.f.



Conclusions

✓ **Oceanic errors :** inhomogeneous, anisotropic and non-stationary ; fast evolution of errors (~24h) strongly correlated to atm. error development.

✓ **Control of the model errors :**

- encouraging performances of the reduced-order data assimilation system
- need to correct both atmospheric and oceanic variables
- the time-dependency of errors should be considered

Perspectives

✓ real data experiment
✓ consider the atm. field resolution error in our study (with ALADIN, AROME f.i.)
✓ improve the prediction range length :

- localize the correction
- enhance the OI scheme (f.i. improve the time-dependency of error EOFs)
- implement a more complex/costly scheme such as Reduced-Order Ensemble Kalman Filter or Ensemble Kalman Filter

✓ multiple twin-experiment

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