Using Empirical Mode Decomposition to Calculate Patterns of Global Sea Level Change

Benjamin Hamlington, Robert R. Leben, R. Steven Nerem
Colorado Center for Astrodynamics Research
University of Colorado at Boulder

GLOBAL SEA LEVEL CHANGE

Long-term mean sea level change has become a topic of considerable interest in studies of global climate change. Measurements of changes in global sea level height can provide validation for predictions made by climate models of global warming. Through the use of empirical mode decomposition (EMD) and empirical orthogonal function analysis (EOF), estimates of both the rate of change of mean sea level and the acceleration of this rate have been made. The EMD method has the advantage of being direct and adaptive, with an *a posteriori*-defined basis derived solely from the data. When combined with EOF analysis, we are able to obtain estimates for both the spatial and temporal variation of the rate of sea level rise. To calculate the acceleration, we first use an optimal derivative filter on the global height field. The EMD is performed on the resulting differenced dataset, and again using EOF analysis, we can determine the spatial and temporal variation of the acceleration.

EMPIRICAL MODE DECOMPOSITION METHOD

The empirical mode decomposition method (EMD) is based on the assumption that any data consists of different, simple intrinsic modes of oscillation. The EMD process allows higher frequency modes to be "sifted" out until only a strictly increasing or strictly decreasing function remains. The sifting process is completed as follows (*Huang*):

1) An upper envelope is constructed by fitting a cubic spline to the local maxima of the data set (Figure 1-top).

2) A lower envelope is constructed by fitting a cubic spline to the local minima of the data set (Figure 1-top).

3) The mean, m_1, of the upper and lower envelopes is calculated and subtracted from the original data (Figure 1- bottom):

$$\mathbf{h_1 = x(t) - m_1.}$$

4) h_1 is then treated as the original data and steps 1 through 3 are repeated (Figure 1- bottom).

$$\mathbf{h_{1k} = h_{1(k-1)} - m_1.}$$

5) This process continues until the squared difference of two successive sifting operations is less than some predetermined value. Once the difference between successive iterations is small, h_{1k} is said to be the first intrinsic mode function (IMF_1).

6) The first IMF is then separated from the original data:

$$\mathbf{R_1 = x(t) - IMF_1.}$$

If local maxima and minima still exist, the sifting process is repeated on $\mathbf{R_1}$ to find the next IMF.

EMPIRICAL MODE DECOMPOSITION - SIFTING PROCESS

As mentioned above, the sifting process is completed by fitting cubic splines through the local maxima and minima of the dataset. Using variational techniques, it is easy to show that a natural spline (second derivative set equal to zero at the endpoints) minimizes the curvature of the fitted spline, and thus is used for the EMD. The mean of the upper and lower splines is computed and subtracted from the data. The process repeats until the squared difference between successive iterations is acceptably small. The iterative process is described in steps 3 through 5 above.

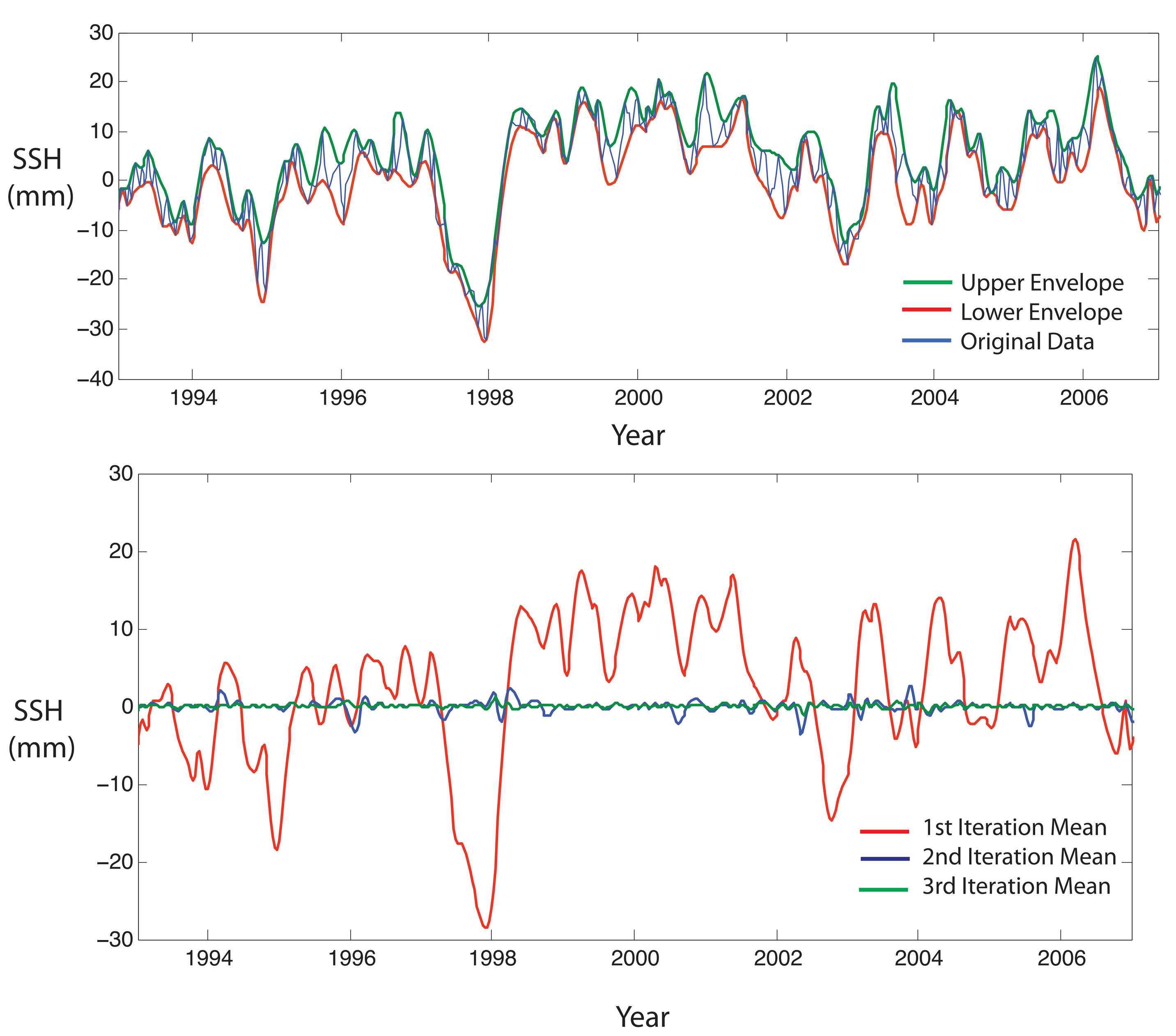


Figure 1. Top figure shows construction of upper and lower envelopes using cubic splines. Original data is shown in blue. Bottom figure shows mean of upper and lower envelopes calculated by successive iterations of the sifting process.

EXAMPLE OF EMPIRICAL MODE DECOMPOSITION

In order to calculate the patterns of global sea level change, we first must perform empirical mode decomposition on each 1x1 degree window across the globe. Figure 2 below shows the result from the EMD of a single window. The 6 plots are called the intrinsic mode functions (IMFs) of the original data. Note that the trend in the data is given by the last IMF.

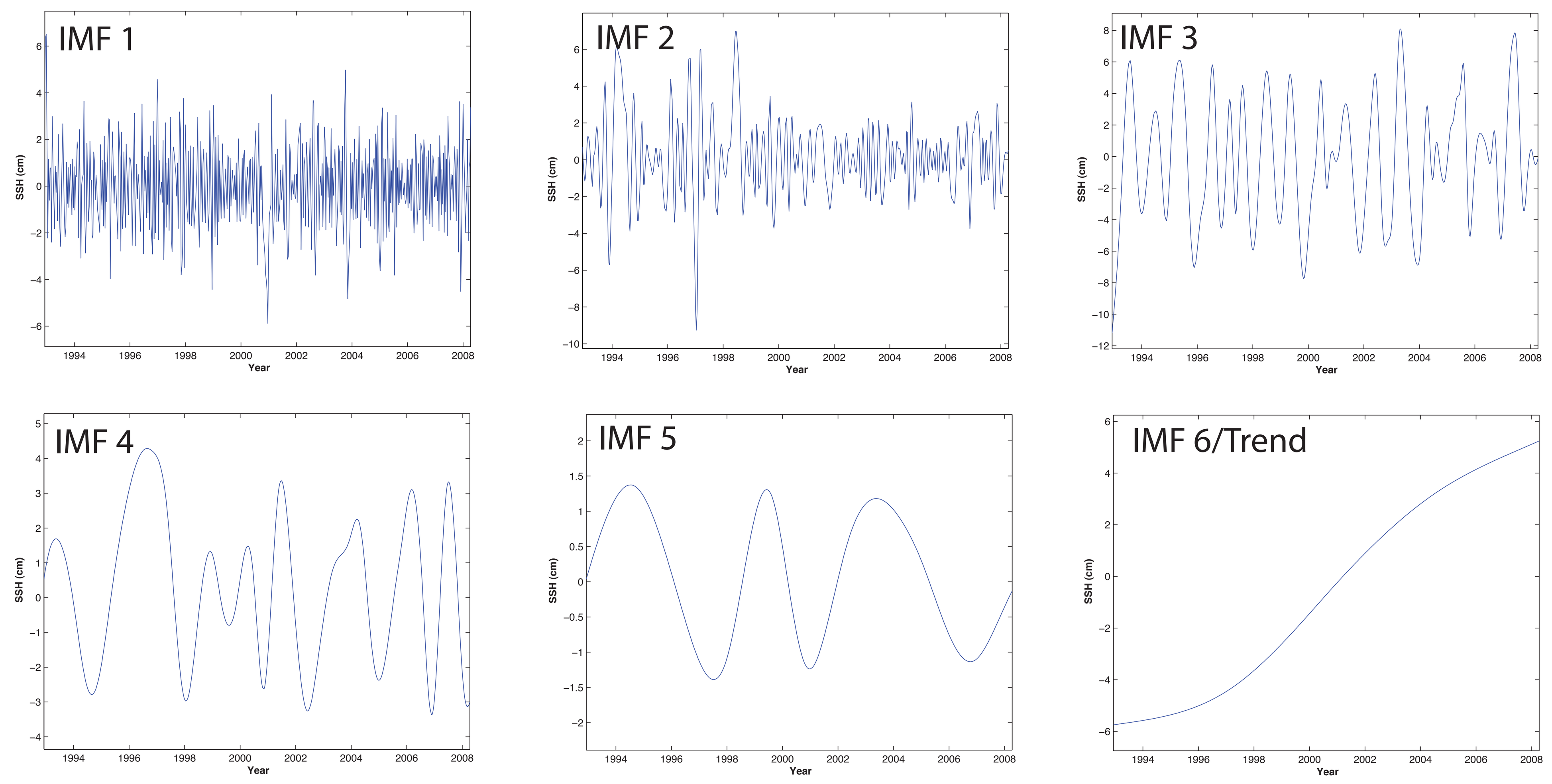


Figure 2. Empirical mode decomposition of sea surface height for a single 1x1 degree window over entire time series. The sum of the 6 IMFs will return the original data.

SPATIAL AND TEMPORAL VARIATION OF SEA LEVEL RISE

Using the EMD method and EOF analysis, spatial and temporal patterns of the sea level rise are obtained. Altimetry data obtained from a combination of TOPEX/POSEIDON (T/P) and Jason-1 is used for this analysis. The process is completed as follows:

1) Perform EMD on each 1x1 degree window of sea surface height anomaly data.
2) Form a new matrix with columns given by the residual (trend) obtained from the EMD at each window.
3) Perform EOF analysis on this matrix to obtain the spatial and temporal patterns.

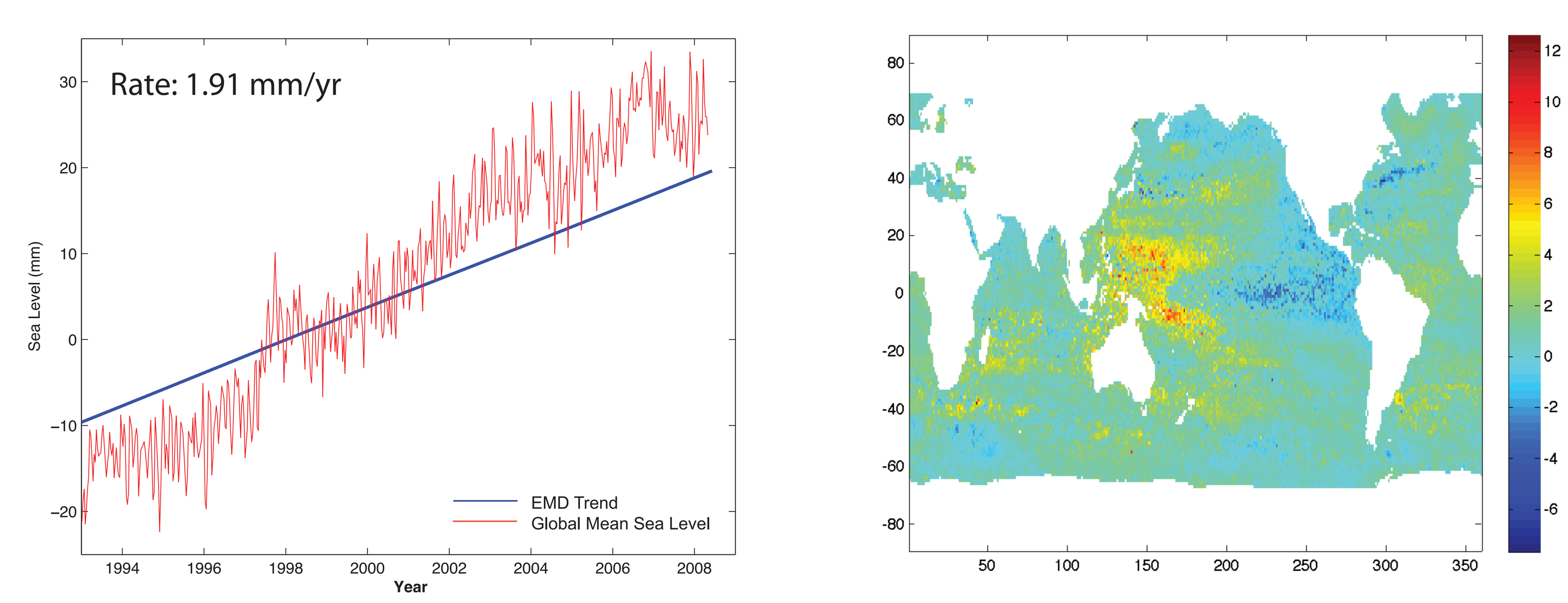


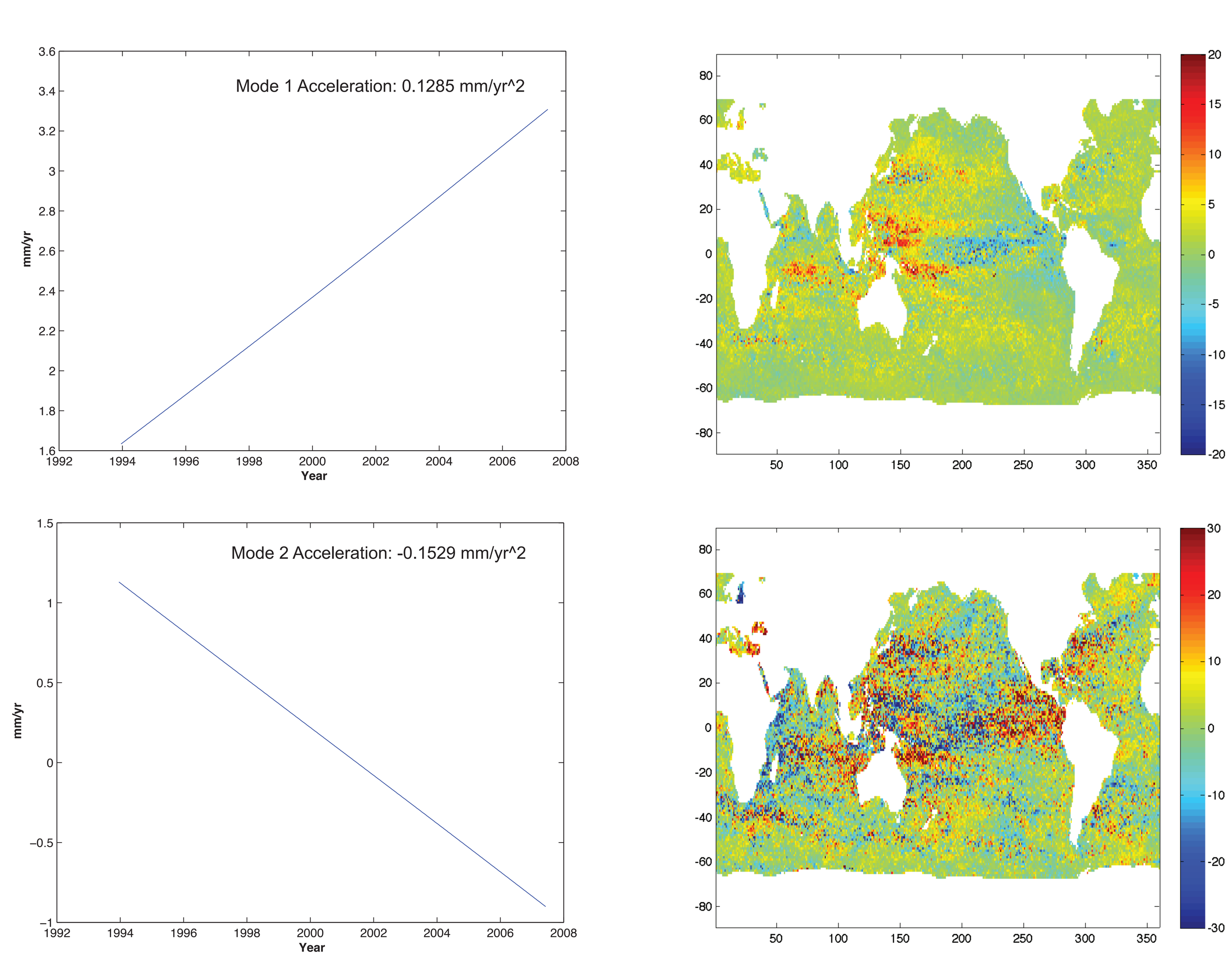
Figure 3. Global mean sea level rise time series and spatial map, as calculated from EMD and EOF analysis. As estimated from the time series, the rate of global sea level rise is estimated to be 1.91 mm/yr.

SPATIAL AND TEMPORAL VARIATION OF SEA LEVEL ACCELERATION

To calculate the acceleration of the sea level rise, we used a Savitzky-Golay derivative filter on the original sea surface height dataset to obtain a differenced dataset. We then completed the same 3 steps as outlined in the section above to obtain the spatial and temporal variation of the sea level acceleration.

By varying the length of the time series used in the decomposition and analysis, we found that the calculated acceleration was very sensitive to shorter scale temporal features. A dataset from 1993 to the middle of 2007 yielded an acceleration of the global sea level rise, while a dataset composed of measurements taken between 1993 and the middle of 2008 produced a deceleration. This suggests that the available time series is too short to accurately compute an acceleration, but does not detract from the ability of the EMD method to withdraw an estimate of the acceleration from the sea level height dataset.

Figure 4. Global mean sea level acceleration time series and spatial map, as calculated from EMD and EOF analysis. The differenced dataset was obtained using a Savitzky-Golay filter with a 1 year half window width. The time range of the data used was from 1993 to the middle of 2008. Top two figures are the time series and spatial map for the 1st component yielded by the EOF analysis. The bottom two figures are the time series and spatial map for the 2nd component. Combining the trends gives a global sea level deceleration of 0.024 mm/yr^2.



References: Huang, N.E., Shen, S.S. 2005. *Hilbert-Huang Transform and Its Application.* Interdisciplinary Mathematical Sciences- Vol 5.