Effect of Meridional Shear on Equatorial Waves - Revisited

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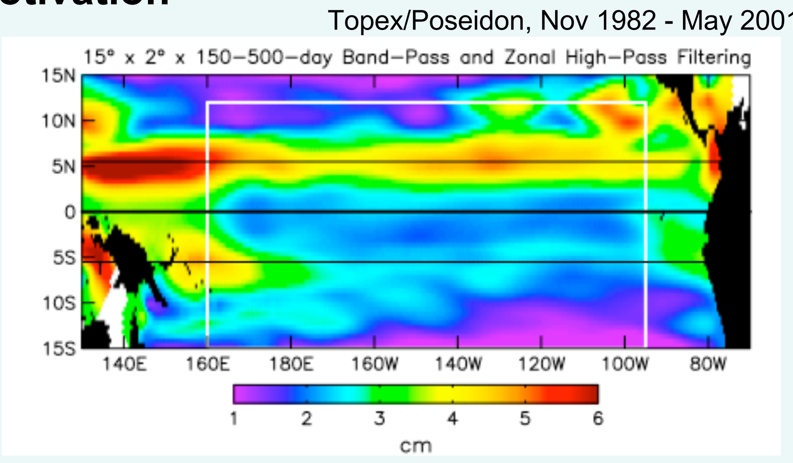
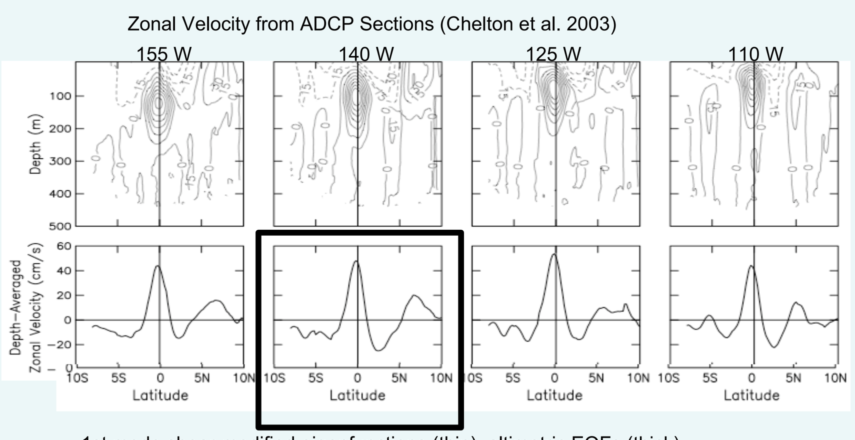
Motivation

SSH Standard Deviation
Topex/Poseidon, Nov 1982 - May 2001

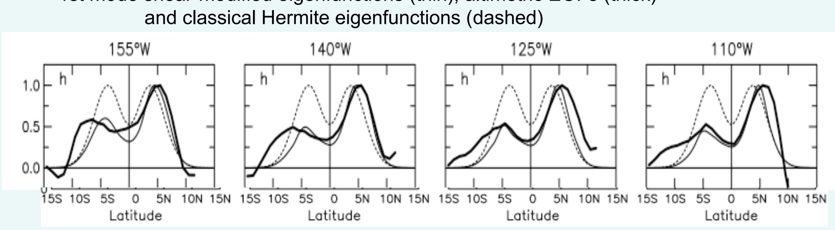


Chelton et al. 2003 showed that the quasi-annual Rossby wave is highly asymmetric about the equator across much of the equatorial Pacific. They also showed that when the shallow water equations are linearized about a background current system representative of the upper 250m, the numerically calculated ssh eigenfunctions of the first meridional mode Rossby wave bear a striking resemblance to the corresponding first eofs of altimetry observations.

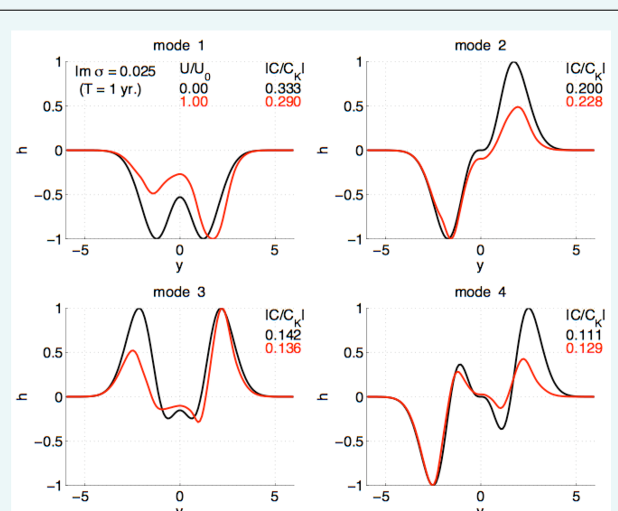
Zonal Velocity from ADCP Sections (Chelton et al. 2003)



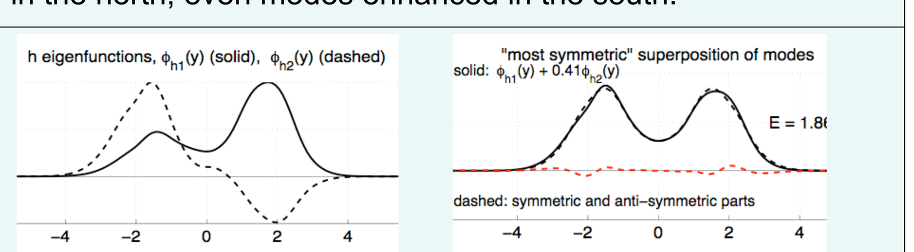
1st mode shear-modified eigenfunctions (thin), altimetric EOFs (thick) and classical Hermite eigenfunctions (dashed)



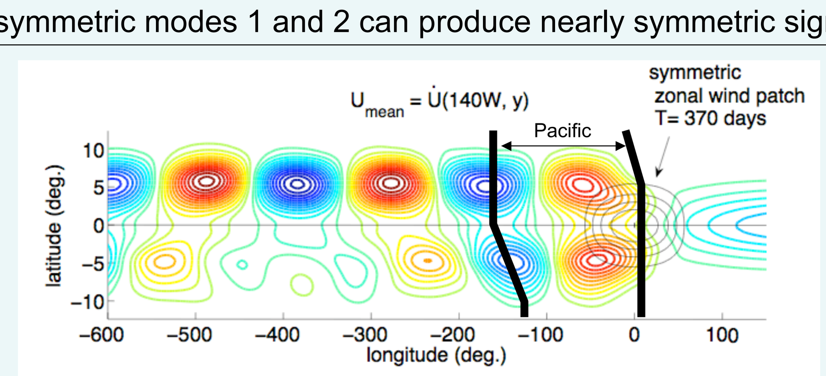
In spite of the remarkable coincidence noted above, forcing patterns must still be invoked to explain observed asymmetry, because the asymmetries introduced in higher meridional modes can compensate for the asymmetry in the first mode.



Lowest 4 meridional modes for annual Rossby wave ssh
No background currents (black), 140W equatorial current system (red)
Note the complementary asymmetries: odd modes enhanced in the north, even modes enhanced in the south.



Asymmetric modes 1 and 2 can produce nearly symmetric signal.

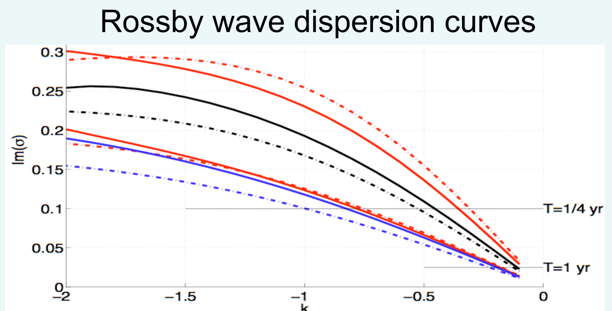


SSH in 1-1/2 layer model linearized about U(140W,y). Symmetric zonal wind produces locally symmetric response (modes 1& 2). At annual period, modes can't separate in time to produce significant asymmetry within Pacific basin.

Rossby wave dispersion curves

No background currents: dashed

140W equatorial currents: solid.



Modification of Rossby wave dispersion relations by equatorial currents: lowest four meridional modes. Difference in phase speed between odd and even modes is decreased.

Research Goal

In order to use equatorial wave theory effectively to explain the observations, we first seek a comprehensive understanding of how the equatorial current system affects the equatorial wave spectrum:

- Source(s) of dispersion and structure modification?
  - (Advection, background pv anomalies, layer thickness anomalies)
- Why are different modes affected differently?
- Is complementary modification of mode structures/dispersion relations inevitable or fortuitous?

Perturbation expansion solution

After Ripa and Marinone 1983

Shallow water equations linearized about weak, geostrophic, mean zonal currents

$$u = u_*/c = \hat U(y) + \varepsilon \hat u(x,y,t), \qquad c = \sqrt{g' H_0},$$
$$v = v_*/c = \varepsilon \hat v(x,y,t), \qquad (x,y) = (x_*, y_*)\sqrt{\beta / c},$$
$$h = h_*/H_0 = 1 + \hat H(y) + \varepsilon \hat h(x,y,t), \qquad t = t_*\sqrt{\beta c}.$$

$$\partial_t \hat u + \hat U \partial_x \hat u + (\partial_y \hat U - y)\hat v + \partial_x \hat h = 0,$$
$$\partial_t \hat v + \hat U \partial_x \hat v + y \hat u + \partial_y \hat h = 0, \qquad (\hat U, \hat H) = \varepsilon (U, H),$$
$$\partial_t \hat h + \hat U \partial_x \hat h + (1+\hat H)\partial_x \hat u + \partial_y[(1+\hat H)\hat v] = 0. \qquad (\hat u, \hat v, \hat h) = \varepsilon (u,v,h) e^{i(kx - \sigma t)}.$$

$$\left[\begin{pmatrix} 0 & -y & ik \\ y & 0 & \partial_y \\ ik & \partial_y & 0 \end{pmatrix} + \varepsilon \begin{pmatrix} ikU & \partial_y U & 0 \\ 0 & ikU & 0 \\ ikH & (\partial_y H + H\partial_y) & ikU \end{pmatrix} - i\sigma \right]\begin{pmatrix} u \\ v \\ h \end{pmatrix} = 0$$

$$\begin{pmatrix} u \\ v \\ h \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} u^n \\ v^n \\ h^n \end{pmatrix}\varepsilon^n, \qquad \sigma = \sum_{n=0}^{\infty} \sigma_n \varepsilon^n, \qquad \left[\begin{pmatrix} 0 & -y & ik \\ y & 0 & \partial_y \\ ik & \partial_y & 0 \end{pmatrix} - i\sigma_0\right]\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} = 0.$$

$O(1)$: Classical Hermite solutions

$$(L_k - i\sigma_0)\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} = 0, \qquad \begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \tilde u_{m,n} \\ \tilde v_{m,n} \\ \tilde h_{m,n} \end{pmatrix}, \quad \sigma_0 = \tilde\sigma_{m,n}, \quad \begin{pmatrix} m: \text{meridional mode \#} \\ n = 1 \quad for\ n = -1\ (Kw) \\ n = 1,3 \quad for\ n = 0\ (Yw) \\ n = 1,2,3\ for\ n > 0\ (Rw\ \&\ Igw) \end{pmatrix}$$

Normalization: $\int \left(|\tilde u_{m,n}|^2 + |\tilde v_{m,n}|^2 + |\tilde h_{m,n}|^2 \right) dy = 1$

$O(\varepsilon)$ correction:

$$(L_k - i\sigma_0)\begin{pmatrix} u^1 \\ v^1 \\ h^1 \end{pmatrix} = -\left[ikU + \begin{pmatrix} 0 & \partial_y U & 0 \\ 0 & 0 & 0 \\ 0 & \partial_y H & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ ikH & H\partial_y & 0 \end{pmatrix} - i\sigma_1 \right]\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix}$$

Doppler shift — Background pv anomaly — Layer thickness anomaly — Frequency correction

Corrections expanded
In Hermite functions

$$\begin{pmatrix} u^1 \\ v^1 \\ h^1 \end{pmatrix} = \sum A_{p,q} \begin{pmatrix} \tilde u_{p,q} \\ \tilde v_{p,q} \\ \tilde h_{p,q} \end{pmatrix}, \quad A_{m,n} = 0.$$

frequency correction

$$\sigma_1 = -i \int (u^0, v^0, h^0)^* \begin{pmatrix} ikU & \partial_y U & 0 \\ 0 & ikU & 0 \\ ikH & H\partial_y + \partial_y H & ikU \end{pmatrix}\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} dy$$

Symmetric and anti-symmetric parts of background current system

$$\begin{pmatrix} U_S \\ H_S \end{pmatrix} = \begin{pmatrix} U(y) + U(-y) \\ H(y) + H(-y) \end{pmatrix}/2 \qquad \begin{pmatrix} U_A \\ H_A \end{pmatrix} = \begin{pmatrix} U(y) - U(-y) \\ H(y) - H(-y) \end{pmatrix}/2$$

$$\int (u^0, v^0, h^0)^* \begin{pmatrix} ikU_A & \partial_y U_A & 0 \\ 0 & ikU_A & 0 \\ ikH_A & (\partial_y H_A + H_A \partial_y) & ikU_A \end{pmatrix}\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} dy = 0$$

$$\text{so} \quad \sigma_1 = -i \int (u^0, v^0, h^0)^* \begin{pmatrix} ikU_S & \partial_y U_S & 0 \\ 0 & ikU_S & 0 \\ ikH_S & (\partial_y H_S + H_S \partial_y) & ikU_S \end{pmatrix}\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} dy$$

Dispersion relation only affected by symmetric part of (U,H)

$$\begin{pmatrix} u^1_{p+} \\ v^1_{p-} \\ h^1_{p+} \end{pmatrix} = -(L - i\sigma_0)^{-1}\left[ikU_S + \begin{pmatrix} 0 & \partial_y U_S & 0 \\ 0 & 0 & 0 \\ ikH_S & (\partial_y H_S + H_S\partial_y) & 0 \end{pmatrix} - i\sigma_1 \right]\begin{pmatrix} u^0_{p+} \\ v^0_{p-} \\ h^0_{p+} \end{pmatrix}$$

$$\begin{pmatrix} u^1_{p-} \\ v^1_{p+} \\ h^1_{p-} \end{pmatrix} = -(L - i\sigma_0)^{-1}\left[ikU_A + \begin{pmatrix} 0 & \partial_y U_A & 0 \\ 0 & 0 & 0 \\ ikH_s & (\partial_y H_A + H_A\partial_y) & 0 \end{pmatrix} - i\sigma_1 \right]\begin{pmatrix} u^0_{p+} \\ v^0_{p-} \\ h^0_{p+} \end{pmatrix}$$

Asymmetry in eigenfunctions produced by antisymmetric part of (U,H)

Phase speed modification

(In Progress)

$$\frac{\sigma_1}{\sigma_0} = \frac{k}{\sigma_0}\int dy\, U_S \left(|u^0|^2 + |v^0|^2 + |h^0|^2 \right)$$
A: Advection / Doppler shift

$$+ \frac{1}{2(k^2 - \sigma_0^2)}\frac{k}{\sigma_0}\int dy\, (v^0)^2 \partial_{yy} U_S$$
B1: Projection of meridional velocity onto gradient of background p.v. anomaly: $\partial_{yy} U_s$

$$+ \frac{1}{2(k^2 - \sigma_0^2)}\frac{k}{\sigma_0}\int dy\, (v^0)^2 \partial_y (y H_S)$$
B2: Ditto for layer thickness contribution to p.v. anomaly

$$- \frac{k}{\sigma_0}\int dy\, H_S h^0 u^0$$
C: Thickness anomaly?

$$+ \int dy\, H_S (v^0)^2$$
D: Thickness anomaly?

$$- \frac{1}{k^2 - \sigma_0^2}\int dy\, (v^0)^2 y\, \delta Q_S$$
E: ?

$\delta Q \equiv -(\partial_y U + yH)$ Is the linearized background pv anomaly.

$O(\varepsilon)$ correction to long-Rossby wave frequency (phase speed) due to background current $\varepsilon U = \hat U(140W, y)$

Fractional contributions of various terms

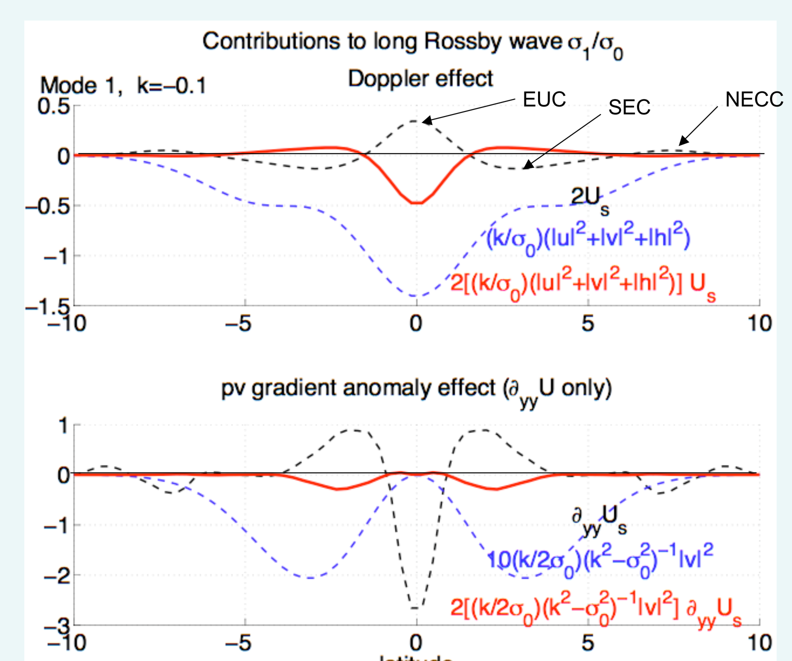
mode	$\varepsilon\sigma_1/\sigma_0$	A Doppler	B1 $v^2\partial_{yy}\hat U$	B2 $v^2\partial_y(y\hat H)$	C $-\hat H hu$	D $\hat H v^2$	E $v^2 y\,\delta\hat Q$
1	-0.25	0.27	0.59	+0.08	+0.11	+0.00	-0.05
2	+0.21	0.50	0.51	-0.06	-0.06	+0.00	+0.11
3	-0.08	0.68	0.74	-0.11	-0.19	-0.00	-0.12
4	+0.11	0.02	0.67	+0.13	+0.16	+0.00	+0.03

For lower modes, the Doppler shift and pv modification by the curvature in the background current are the dominant effects.

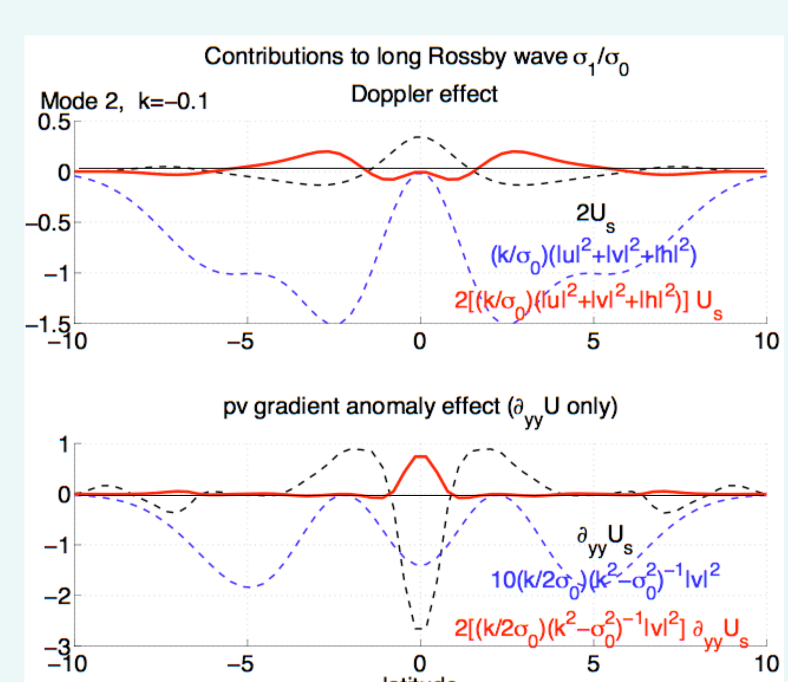
$(\Delta\sigma/\sigma_0)/\alpha$ from numerical eigenvalue solution for k=-0.1
$\hat U = \alpha\, \hat U(140W, y)$

mode	σ_0	$\alpha = 0.3$	$\alpha = 1.0$
1	0.033	-0.22	-0.13
2	0.020	+0.19	+0.14
3	0.014	-0.09	-0.05
4	0.011	+0.13	+0.16

First order correction term matches numerical solution reasonably well when $\hat U = 0.3\,\hat U(140W, y)$. Higher order terms important for full-strength current.



Mode 1 advected primarily by Equatorial Undercurrent, and also slowed down by negative pv-gradient anomaly produced by positive Uyy on flanks of undercurrent.



Mode 2 advected primarily by South Equatorial Current, and also sped up by positive pv-gradient anomaly produced by negative Uyy at peak of undercurrent.

Eigenfunction modification

(In progress)

Expressed in terms of Hermite solutions:

$$\begin{pmatrix} u^0 \\ v^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \tilde u_{mn} \\ \tilde v_{mn} \\ \tilde h_{mn} \end{pmatrix}, \quad \sigma^0 = \tilde\sigma_{mn} \qquad \begin{pmatrix} u^1 \\ v^1 \\ h^1 \end{pmatrix} = \sum_{p,q} A_{pq} \begin{pmatrix} \tilde u_{pq} \\ \tilde v_{pq} \\ \tilde h_{pq} \end{pmatrix}$$

Expansion coefficients:

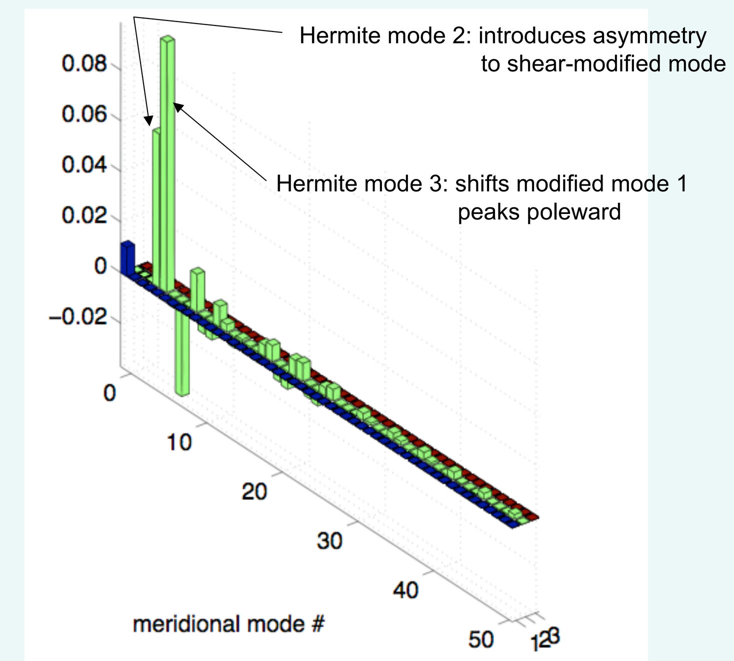
$$A_{pq} = \frac{i}{\tilde\sigma_{pq} - \tilde\sigma_{mn}} \int \left(\tilde u^*_{pq}\ \ \tilde v^*_{pq}\ \ \tilde h^*_{pq} \right) \begin{pmatrix} ikU & \partial_y U & 0 \\ 0 & ikU & 0 \\ ikH & H\partial_y + \partial_y H & ikU \end{pmatrix}\begin{pmatrix} \tilde u_{mn} \\ \tilde v_{mn} \\ \tilde h_{mn} \end{pmatrix} dy$$

or

$$A_{pq} = \frac{-k}{\tilde\sigma_{pq} - \tilde\sigma_{mn}} \int U \left(\tilde u^*_{pq}\ \ \tilde v^*_{pq}\ \ \tilde h^*_{pq} \right)\begin{pmatrix} \tilde u_{mn} \\ \tilde v_{mn} \\ \tilde h_{mn} \end{pmatrix} dy \qquad \text{Advection}$$

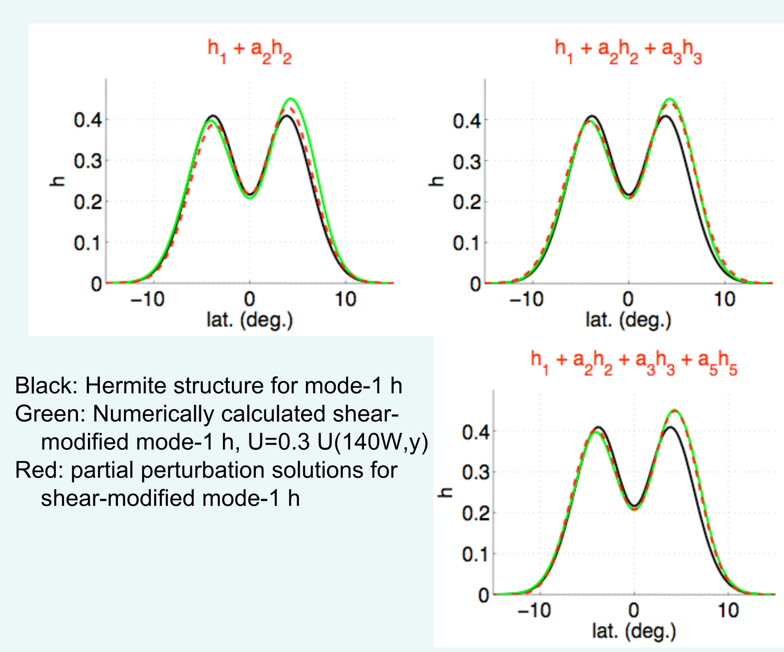
$$+ \frac{\tilde\sigma_{mn}}{\tilde\sigma_{pq} - \tilde\sigma_{mn}} \int H\, \tilde h_{pq}\, \tilde h_{mn}\, dy \qquad \text{Thickness anomaly}$$

$$- \frac{i}{\tilde\sigma_{pq} - \tilde\sigma_{mn}} \int \partial_y U\, \tilde u_{pq}\, \tilde v_{mn}\, dy$$
$$- \frac{i}{\tilde\sigma_{pq} - \tilde\sigma_{mn}} \int \partial_y H\, \tilde h_{pq}\, \tilde v_{mn}\, dy \qquad \text{pv anomalies?}$$



Expansion coefficient spectrum for correction to mode 1 Rossby wave eigenfunctions
U(y) = 0.3 U(140W,y), k=-0.1
contributing modes:
Blue: Kelvin and eastward Yanai and inertia-gravity modes
Green: Rossby modes
Red: westward Yanai and inertia-gravity modes

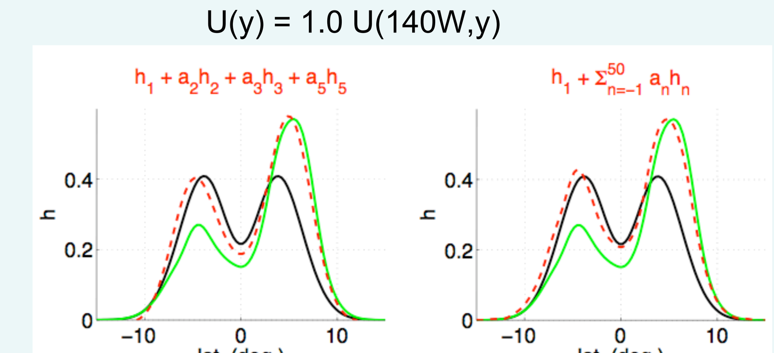
Modifications to mode-1 long Rossby wave h structure
U(y) = 0.3 U(140W,y)



Black: Hermite structure for mode-1 h
Green: Numerically calculated shear-modified mode-1 h, U=0.3 U(140W,y)
Red: partial perturbation solutions for shear-modified mode-1 h

$O(\varepsilon)$ solution accurately predicts eigenstructure for weak currents (30% of nominal). Mode-2 contribution introduces asymmetry, Mode-3 widens eigenfunction & moves peaks poleward.

U(y) = 1.0 U(140W,y)



Higher order solution is required to accurately predict structure and phase speed for full-strength currents. $O(\varepsilon)$ solution correctly predicts trends.
Empirically derived expansion coefficients (right) show that eigenfunction modified by full-strength currents can still be approximated with minimal number of Hermite modes.