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Errors in Estimating River Discharge from Remote Sensing based on Manning's Equation

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Overview

- The Surface Water and Ocean Topography satellite mission will provide unprecedented mapping of water surface heights, slopes, areal extent, and their changes in time.
- The purpose of this study is to assess the accuracy of indirect streamflow estimates that would likely result from applying SWOT-based measurements in a simple slope-area approach (Manning's equation).
- The slope-area method is considered a first-order method and was developed for use with ground-based observations. SWOT will contribute additional spatial information that is expected to improve these estimates.

Surface Water and Ocean Topography Mission

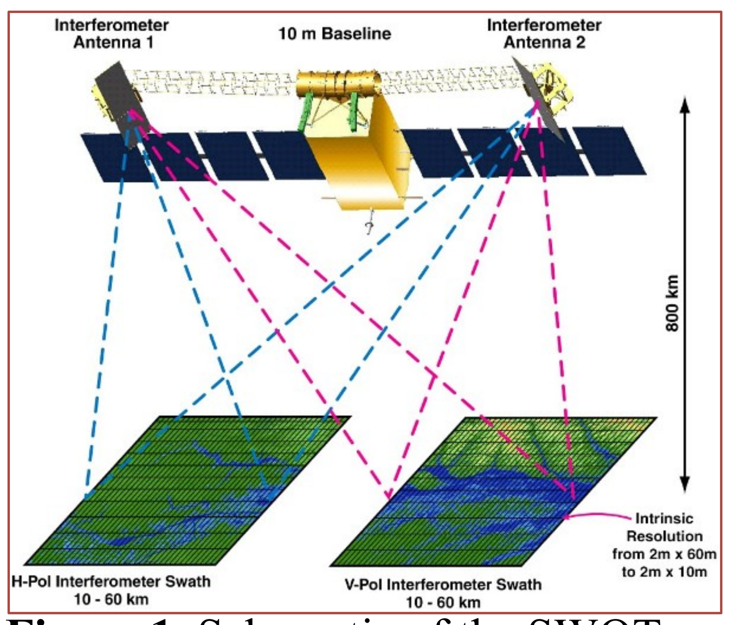


Figure 1. Schematic of the SWOT instrument.

- Ka-band Radar Interferometry (KaRIN).
- Look angles limited to less than 4.5°; 2 60-km wide swaths.
- 22-day repeat cycle, 78° inclination; all rivers, lakes, reservoirs observed at least twice every 22 days.
- Will measure reach-averaged river properties to a high degree of accuracy (Table 1).

Table 1. Requirements for the accuracy of SWOT measurements (Rodríguez, 2009).

Measurement	Required accuracy (1σ)	
Water surface height	10 cm	Averaged over 1 km² area within river mask
Water surface slope	1 cm/ km	Over 10 km downstream distance inside river mask
Water surface areal extent	20%	For all rivers at least 100 m wide

Estimation of Streamflow: Manning's Equation

- Manning's Equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \quad (1)$$

- Assuming rectangular cross-section and width >> depth

$$Q = \frac{1}{n} w z^{5/3} S^{1/2} \quad (2)$$

- In terms of SWOT observables

$$Q = \frac{1}{n} w \left(z_0 + dz\right)^{5/3} S^{1/2} \quad (3)$$

- Assuming uniform flow, S = friction slope ~ water surface slope

Q = discharge (m³/s)
n = Manning's roughness coefficient
A = cross-sectional area of channel (m²)
R = hydraulic radius = A/P (m)
P = wetted perimeter (m)
w = width (m)
z = water depth (m)
z_0 = water depth at some arbitrary "initial" time (m)
dz = h – h_0 (m)
h = water elevation at current observation time (m)
h_0 = water elevation corresponding to z_0 (m)

Derived Quantities

- *Width*: Moller and Rodríguez (2008) estimated errors in width resulting from water coherence time effects (due to wind and turbulence of the water surface). For their worst-case (temporal decorrelation time of 20 ms), width errors (1σ) were roughly 5% over a 1 km long reach, and the mean width bias was between 75 and 10 m for coherence times from 4 to 30 ms, respectively. 10 m was the minimum bias due to pixel size.
- *Initial water depth*: Durand et al. (2010) proposed an algorithm to extract an "initial" water depth based on the kinematic and continuity assumptions applied to Manning's equation. For a test case on the Cumberland River in Ohio, the relative error in depth had a mean of 4.2% and a standard deviation of 11.2%.
- *Roughness*: This is our friction factor. A number of regression schemes have been proposed to estimate this quantity from observations. We have tested these regressions with in situ observations (described in next section) and found that mean errors were ~10% with 20-30% standard deviation. In the Monte Carlo analysis, we use Dingman & Sharma's 1997 regression: $n=0.217w^{-0.173}z^{0.094}S^{0.156}$

Test Data: In Situ Reach-Averaged Observations

Reach-average Value	Mean	Standard Deviation	Minimum	Maximum
Q (m³/s)	1083	9056	0.01	283170
w (m)	131	193	2.9	3870
z (m)	2.39	2.36	0.10	33.00
S	0.0026	0.0052	0.000013	0.0418
n	0.034	0.046	0.008	0.664

Table 2. Summary statistics for 1038 in situ observations of streamflow and coincident hydraulic properties on 103 river reaches used for testing the error propagation. The largest river included is the Amazon River. Compiled by Bjerklie et al. (2003).

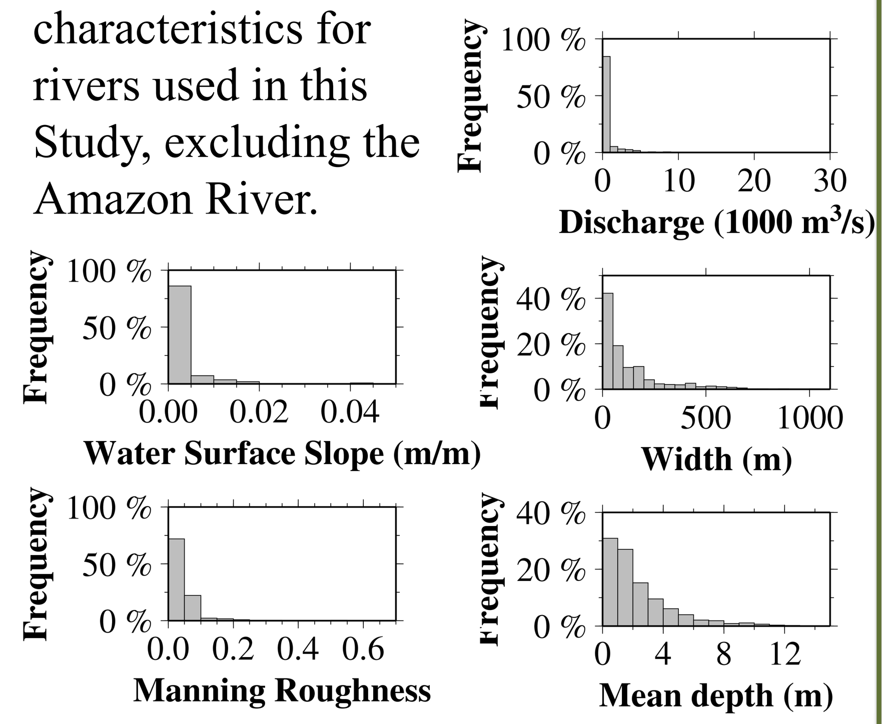


Figure 2. Distributions of hydraulic characteristics for rivers used in this Study, excluding the Amazon River.

First Order Uncertainty Analysis

We assume that Eqn. 3 can be linearized as follows:

$$E[Q(n,w,z,s)] \approx Q(E[n],E[w],E[z],E[s]) \quad (4)$$

$$\therefore Var[Q] \approx ACA^T \quad (5)$$

$$\text{where } A = \left[\frac{\partial Q}{\partial n} \quad \frac{\partial Q}{\partial w} \quad \frac{\partial Q}{\partial z_0} \quad \frac{\partial Q}{\partial h_0} \quad \frac{\partial Q}{\partial h} \quad \frac{\partial Q}{\partial S}\right], \quad (6)$$

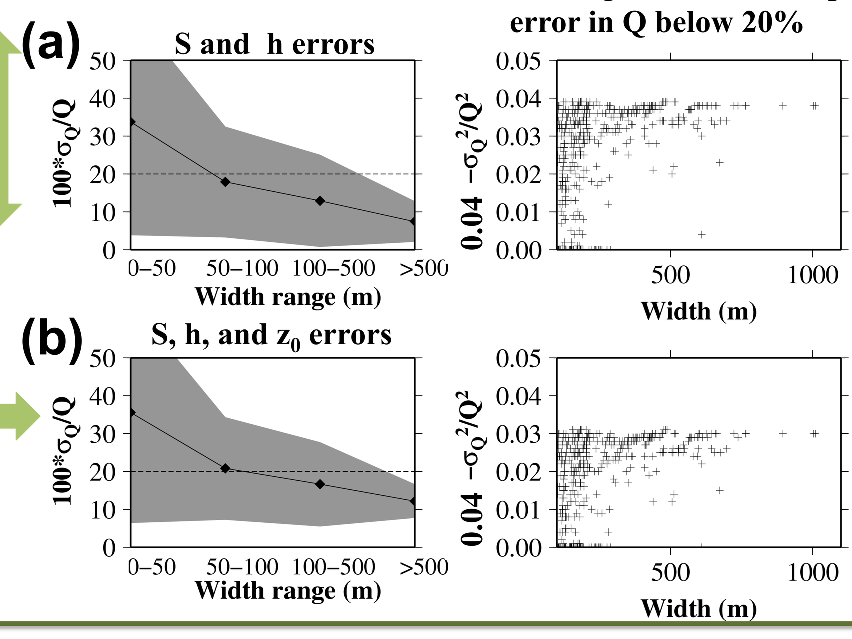
and C is the covariance matrix.
If the terms are assumed to be independent, this becomes:

$$\frac{\sigma_Q}{Q} = \sqrt{\left(\frac{\sigma_n^2}{n^2} + \frac{\sigma_w^2}{w^2} + \frac{25(\sigma_{z_0}^2 + 2\sigma_h^2)}{9z^2} + \frac{\sigma_s^2}{4s^2}\right)} \quad (7)$$

Figure 3. First order uncertainty assuming independent errors, as in Eqn. 6, based on 1038 observations, binned by width. An ideal case of $\sigma_n = 0.1*n$ is used in the part c.

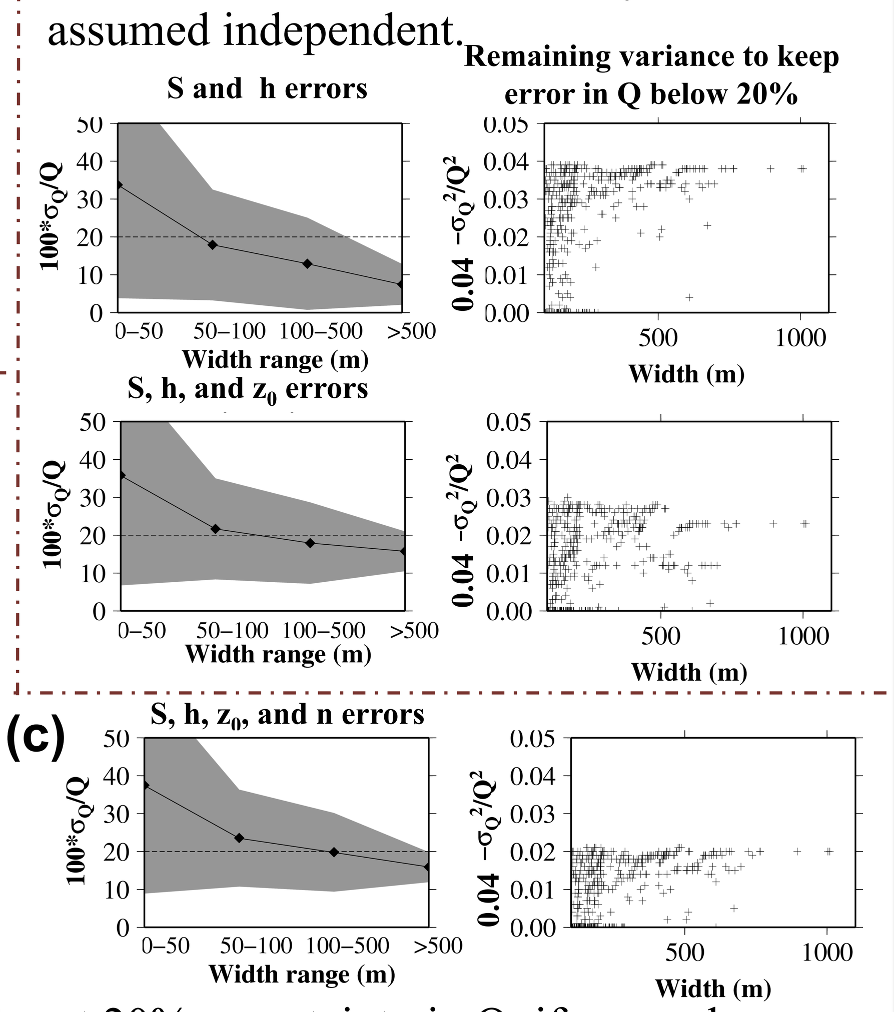
+/- 1 σ^2 in gray bar

Mean at black diamonds



σ_s = 1e-5	σ_h = 0.10 m	σ_{z0} = 0.11*z_0

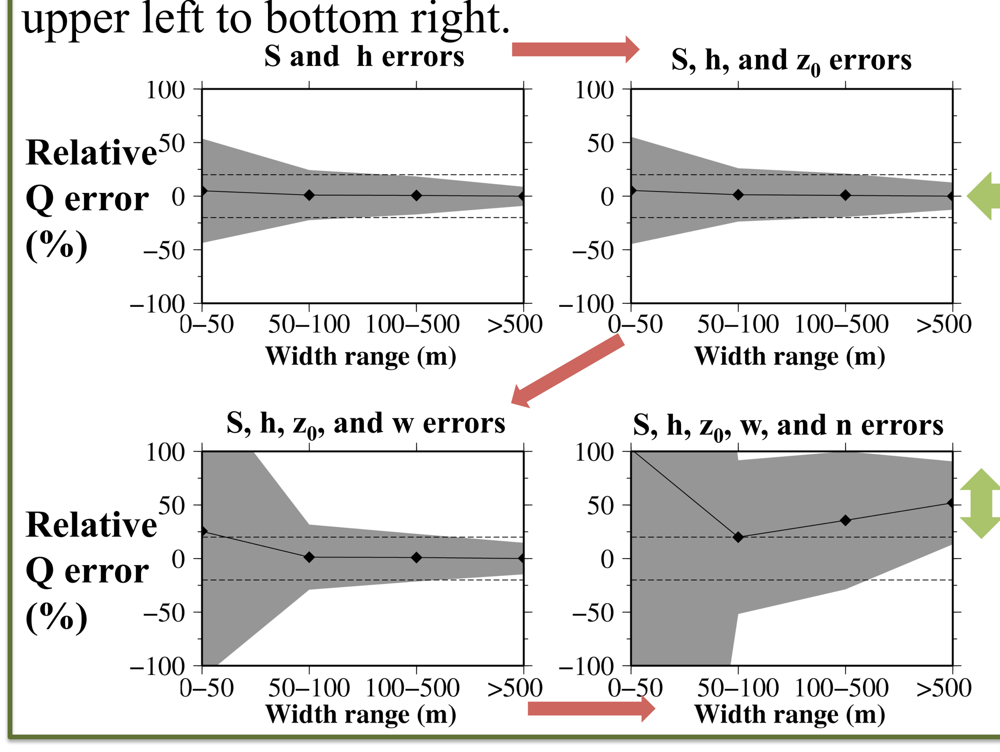
Figure 4. First order uncertainty assuming a correlation of 1.0 between all terms in Eqn. 5, except that errors in h_0 and h were assumed independent.



To get 20% uncertainty in Q: if n or w known perfectly, the other could have at most 17% error (right side b); for 10% error in n, w could have at most 14% error (right side c).

Monte Carlo Estimates of Error

Figure 5. 1000 random perturbations to each observation were generated based on the distributions in box to right (with z_0=0.5z), and Q was calculated by inserting these observations into Manning's equation. Mean and standard deviation of relative error in Q were calculated from all 1,038,000. Errors were progressively added from upper left to bottom right.



- $S^{perturbed}$ = S + N(0,1e-5)
- $z^{perturbed}$ = z + N(0,0.10 m) +N(0,0.1 m)+ z_0*N (0,0.11)
- $w^{perturbed}$ = w + N(0,10 m) + w*N(0,0.07)
- $n^{perturbed}$ calculated from other perturbed values using Dingman and Sharma (1997)

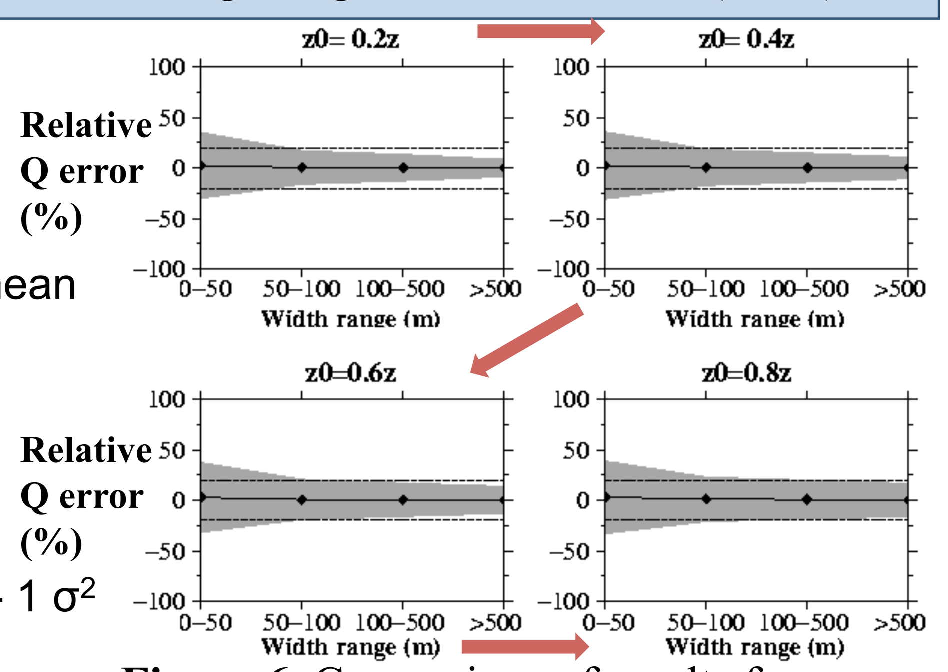


Figure 6. Comparison of results from Monte Carlo with only errors in slope, h, and bathymetry depending on assumed z_0.

Conclusions

- For the base case of Manning's equation for 1-D channel flow, instantaneous discharge can be estimated with accuracies at or near 20% for most rivers wider than 100 m, assuming an improved estimation of n.
- Instantaneous discharge errors in this approach are highly sensitive to errors in total water depth. Estimating depth around low flows would help to limit these errors.
- This analysis depends strongly on the knowledge of error standard deviation and covariance. Additional work is needed to verify and improve estimates of the magnitude of these terms.
- In situ observational errors and the implications of knowledge of spatial extent during times of overbank flow should be considered in future work.
- Future efforts should seek to better understand the correlations between variables. Spatial and temporal sampling combined with continuity and other hydrodynamic assumptions should provide additional constraints not considered here.

Workshop: Towards High-Resolution of Ocean Dynamics and Terrestrial Surface Waters from Space
21-22 October 2010, Lisbon, Portugal