Direct Assimilation of Image Sequences

A. Vidard, I. Souopgui, F.-X. Le Dimet, O. Titaud

Équipe-projet INRIA MOISE, LJK, Grenoble

Arthur.Vidard@imag.fr

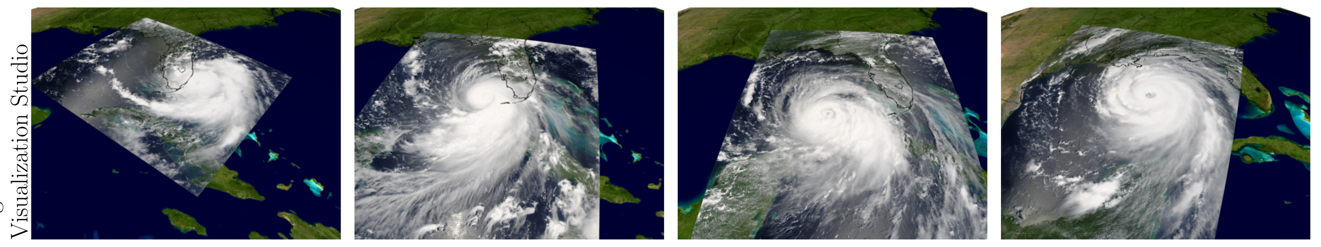
Summary

In order to forecast the evolution of a dynamical system such as gophisical fluids (ocean, atmosphere, continental waters), all the available information have to be accounted for. They are of very different nature : set of non linear PDE (mathematical-type information), in situ measurements and remote sensing (physical-type information), statistical and qualitative informations.
The forecast is produced through a model integration starting from an initial state, from which the system evolution is very sensitive. Consequently, the issue is to evaluate the initial state in a consistent manner from all this heterogeneous sources of information. At the beginning of the 80s, techniques coming from the optimal control theory were proposed to achieve this task. These techniques are now adopted by the main numerical weather forecast centres.
For few decades, a large number satellites dedicated to earth observation has been launched, in order the improve our knowledge of the atmosphere and the oceans. They provide, among other things, numerous sequences of images. These sequences clearly have a strong predictive potential due to the fact that they contain information about the dynamics of the observed system. Currently, this kind of information is unfortunately not used in an optimal manner in conjunction with the numerical models.
This poster presents an extension of the optimal control based techniques to the assimilation of images. A quadratic term measuring the misfit between the images equivalent produced by the model and the observed images is introduced in the usual cost function.

Motivations

Under used, the satellite image sequences have a strong predictive potential: Dynamic evolution of structures from Katrina

NASA/Goddard Space Flight Center Scientific Visualization Studio



25 avril 2008   26 avril 2008   27 avril 2008   28 avril 2008

Evolution of a dry intrusion in a sequence of MeTeOSAT (water vapour canal): from a simple anomaly to cyclogenesis

Santurette et Georgiev 2005

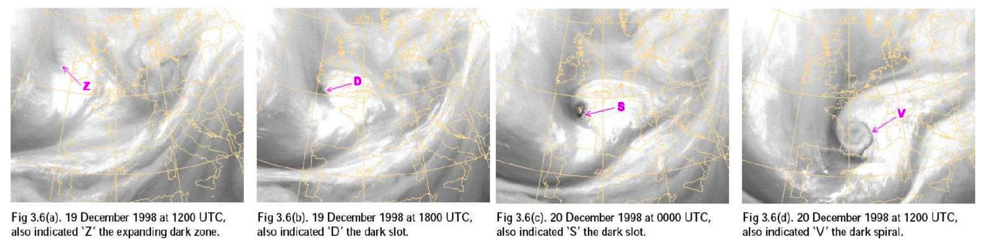


Image sequences contain:

- a large amount of high resolution information,
- structured informations about the dynamics of the observed systems,
- extreme event precursors.

Variational assimilation of image sequences

'Classical' data assimilation

Ingredients

- Model $\begin{cases} \dfrac{\partial x}{\partial t} = F(x) \ , \ t \in [0,T] & (F : \text{Partial Derivative Operator}) \\ x(0) = x_0 & , \text{Initial state} \end{cases}$
  $\rightarrow$ Numerical model: discretized in space and time $(t_i)_{i=0}^N$
  $\mathcal{M}_i(\mathbf{x}_0)$: approximation to the PDE solution at time t_i; with $\mathbf{x}_0$ being the initial state
- Observations: mesures of the state of the system $\mathbf{y}^o$ areavailable
  - Observation operator $\mathcal{H} \in \mathbb{R}^{p\times n}$: from the state variable spate to the observations space.
  - misfit to the observations : $\mathbf{y}^o(t_i) - \mathcal{H}[\mathcal{M}_i(\mathbf{x}_0)]$
- statistics: some statistical information represented by the background and observation error covariance matrices are available.
  - background error covariance matrix $\mathbf{B}$; background $\mathbf{x}_b$ is an a priori estimate of $\mathbf{x}_0$ (from climatologe or a previous forecast)
  - observation error covariance matrix: $\mathbf{R}$

Data assimilation

combine in a consistant manner all the available information about the system during a $[0, T_N]$ window in order to get an analysed optimal inital state $\mathbf{x}_a$ that minimizes the cost function

$$J(\mathbf{x}_0) = \sum_{i=0}^{N} \|\mathbf{y}_i^o - \mathcal{H}[\mathcal{M}_i(\mathbf{x}_0)]\|_{\mathcal{O}}^2 + \|\mathbf{x}_0 - \mathbf{x}_b\|_{\mathcal{X}}^2$$

- $\|\mathbf{x}\|_{\mathcal{X}} = {}^T\mathbf{x}\ \mathbf{B}^{-1}\ \mathbf{x}$: State variable space metric
- $\|\mathbf{x}\|_{\mathcal{O}} = {}^T\mathbf{x}\ \mathbf{R}^{-1}\ \mathbf{x}$: Observation space metric
- Minimization: computation of the cost function's gradient $\nabla_{\mathbf{x}_0} J$. through the resolution of the adjoint equations:
  $\begin{cases} \dfrac{dp}{dt} + \left[\dfrac{\partial F}{\partial x}\right]^T .p = \left[\dfrac{\partial \mathcal{H}}{\partial x}\right]^T [\mathcal{H}[x] - y] \\ P(T) = 0 \end{cases}$

$$\nabla_{\mathbf{x}_0} J = -P(0) + x - x_0$$

(Le Dimet, 1980), (Lewis and Derber, 1985), (Le Dimet et Talagrand, 1986)

Image Sequence Assimilation

Challenge

Define appropriates observation operator and measure of the misfit.

Properties of images et image sequences

- Indirect observations of the system state (radiances in the case of satellite images)
- Pertinent dynamical information localized on singularities
  $\rightarrow$ The misfit to the observation cannot be computed the same way as standard observation.

Proposition

- Define an appropriate space for the computation of the misfit to images.
- Define the appropriate functions mapping the space of model output (resp. the space of image observations) to the above mentionned state.
- New cost function

$$J(x_0) = \underbrace{\int_0^T \|\mathbf{y} - \mathcal{H}[\mathcal{M}_t(x_0)]\|_{\mathcal{O}}^2 dt}_{\text{Classical term } J_o} + \int_0^T \|\underbrace{\mathcal{H}_{\mathcal{V}\to\mathcal{S}}[\mathbf{v}]} - \underbrace{\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathcal{M}_t(x_0)]}\|_{\mathcal{S}}^2 dt + \|x_0 - x_b\|_{\mathcal{X}}^2$$

- $\mathcal{S}$: Misfit to observation space.
- $\mathcal{H}_{\mathcal{V}\to\mathcal{S}}$ Image operator (e.g. multi-scale transformation)
- $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ Image observation operator. Two possible ways:
  * $\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}}$: synthetic images are produced from model output (operator $\mathcal{H}_{\mathcal{X}\to\mathcal{V}}$)
  * Define directly $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ (work in progress ...)

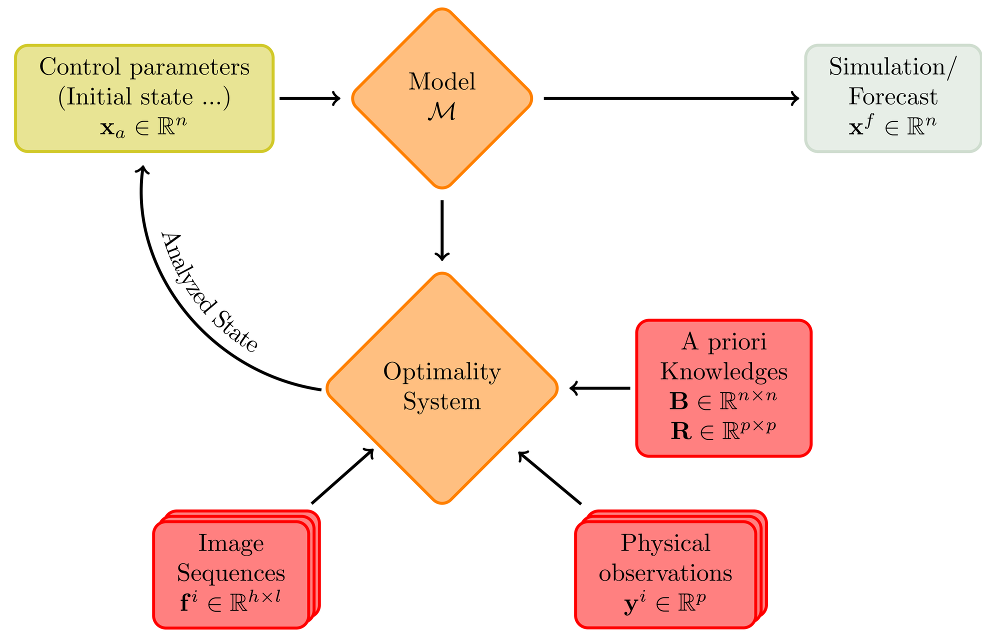
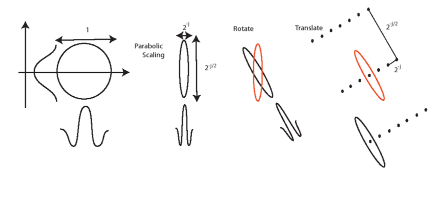


Image Operator

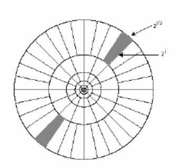
$\mathcal{H}_{\mathcal{V}\to\mathcal{S}}(\mathbf{v})$ = Threshold of the image $\mathbf{v}$ Curvelet Transform

- multi-scale multi-orientation transform
- Decomposition in the curvelet frame: $\mathbf{v} = \sum_{j,k,l} \langle \mathbf{v}, \varphi_{j,l,k}\rangle \varphi_{j,l,k}$

  j : scale index
  l : orientation index (j dependent)
  k : position index (j and l dependent)
- Threshold : $\hat{\mathbf{v}}_m = \sum_{(j,k,l)\in E} \langle \mathbf{v}, \varphi_{j,l,k}\rangle \varphi_{j,l,k}$ # E = m (work in progress ...)

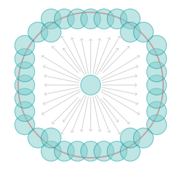


Curvlet principle: scaling, rotating and translating



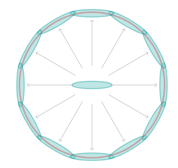
spectral partition of the frequency plan

Wavelets



$\|\mathbf{v} - \hat{\mathbf{v}}_m\| \approx m^{-1}$

Curvlets



$\|\mathbf{v} - \hat{\mathbf{v}}_m\| \approx Cm^{-2}(\log m)^3$

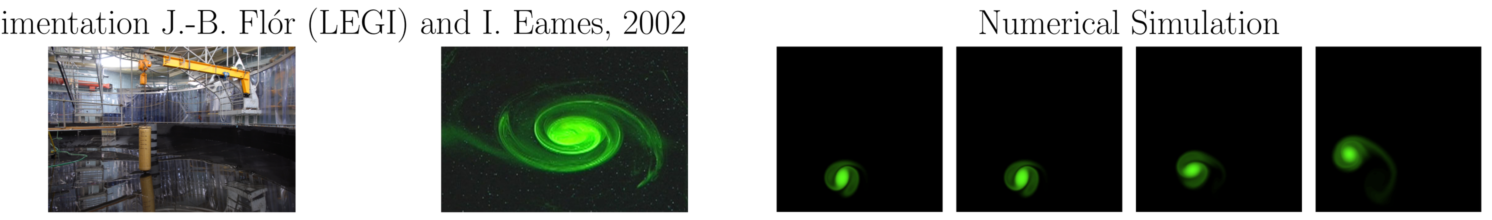
- The curvlet transform is well suited for the compact representation of 2D singularities
  This allows to reduce drastically the size of the observation space (spce where the misfit to the observation is computed)
  $\rightarrow$ speed-up the convergence of the minimization algorithm.
- To a given precision, the curvlet transform requires less coefficients than a wavelet transform for representing a given curve (2D singularities).
- Fast Discrete Curvelet Transform (FDCT) : requires $O(n^2 \log n)$ operations for a $n \times n$ image (`www.curvelet.org`)

(E. J. Candès and D. L. Donoho, 2004), (L. Demanet 2006)

Numerical Experiment : Vortex evolution

Experiments and simulation

Experimentation J.-B. Flór (LEGI) and I. Eames, 2002   Numerical Simulation



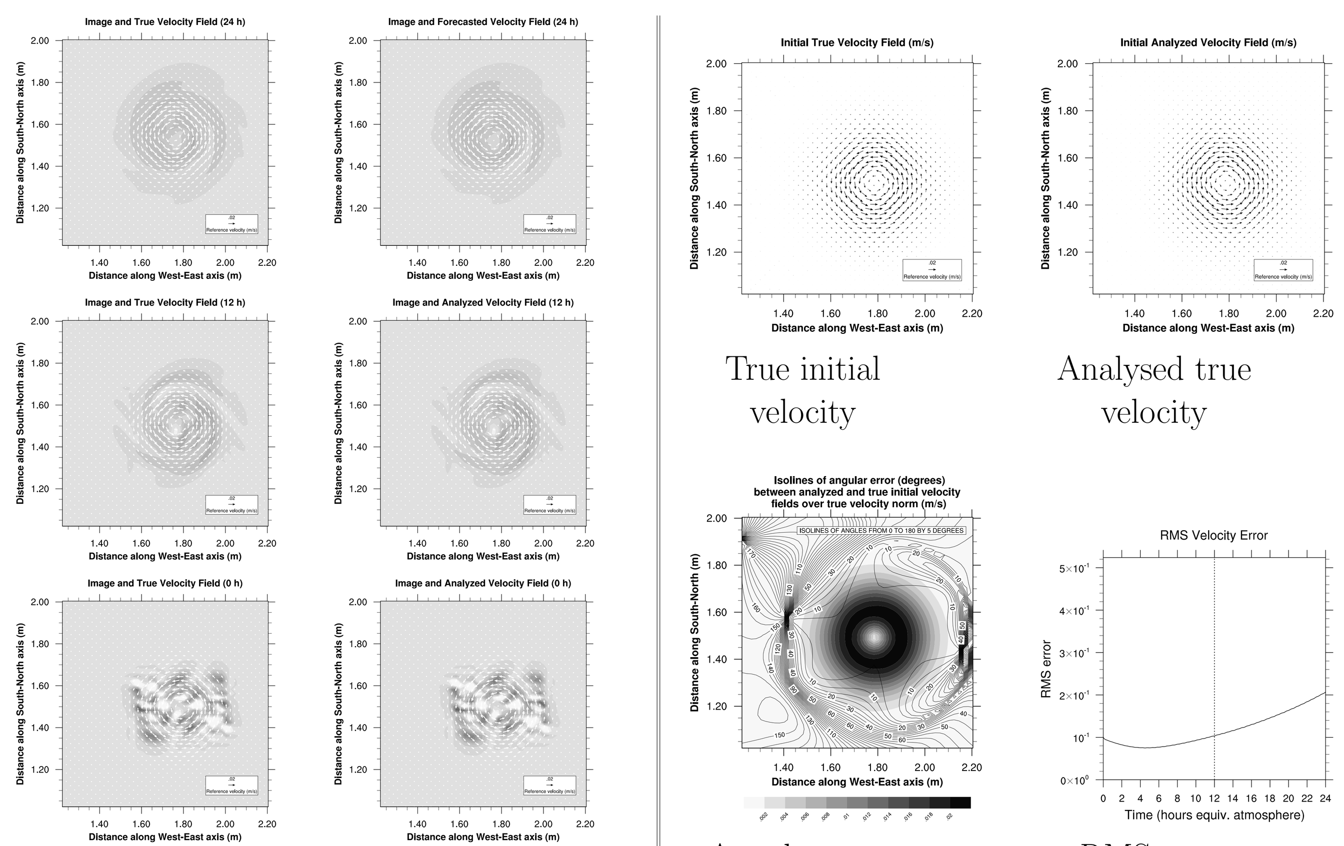
Plateforme Coriolis, LEGI, Grenoble   isolated vortex   Shallow water model, Passive Tracer Advection

Model and Operators State Vector $x = (u, v, h)$; Shallow-water model

$$\begin{cases} \partial_t u - u\partial_x u + v\partial_y u - fv + g\partial_x h + \mathcal{D}(u) = \mathcal{F}_u \\ \partial_t v + u\partial_x v + v\partial_y v + fu + g\partial_y h + \mathcal{D}(v) = \mathcal{F}_v \\ \partial_t h + \partial_x(hu) + \partial_y(hv) = 0 \end{cases}$$

'Image' operator: $\mathcal{H}_{\mathcal{V}\to\mathcal{S}}[\mathbf{v}]$ = threshold of the FDCT$[\mathbf{v}] = \mathcal{T}(\text{FDCT}[\mathbf{v}])$
'Model to images' operator (synthetic image) : $\mathcal{H}_{\mathcal{X}\to\mathcal{V}}[u, v, h] = \mathbf{q}$
where $q(t) = q(x, y, t)$ is the passive tracer concentration transported by the velocity field and verifying $\partial_t q + u\partial_x q + v\partial_y q - \nu_T \Delta q = 0$
Observation operator: $\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}}$ = threshold of FDCT$[\mathbf{q}] = \mathcal{T}(\text{FDCT}[\mathbf{q}])$
Cost function: $J(x_0) = \int_0^T \|\mathcal{T}(FDCT[\mathbf{v}]) - \mathcal{T}(FDCT[\mathbf{q}])\|_{\mathcal{S}}^2 dt + \|x_0 - x_b\|_{\mathcal{X}}^2$

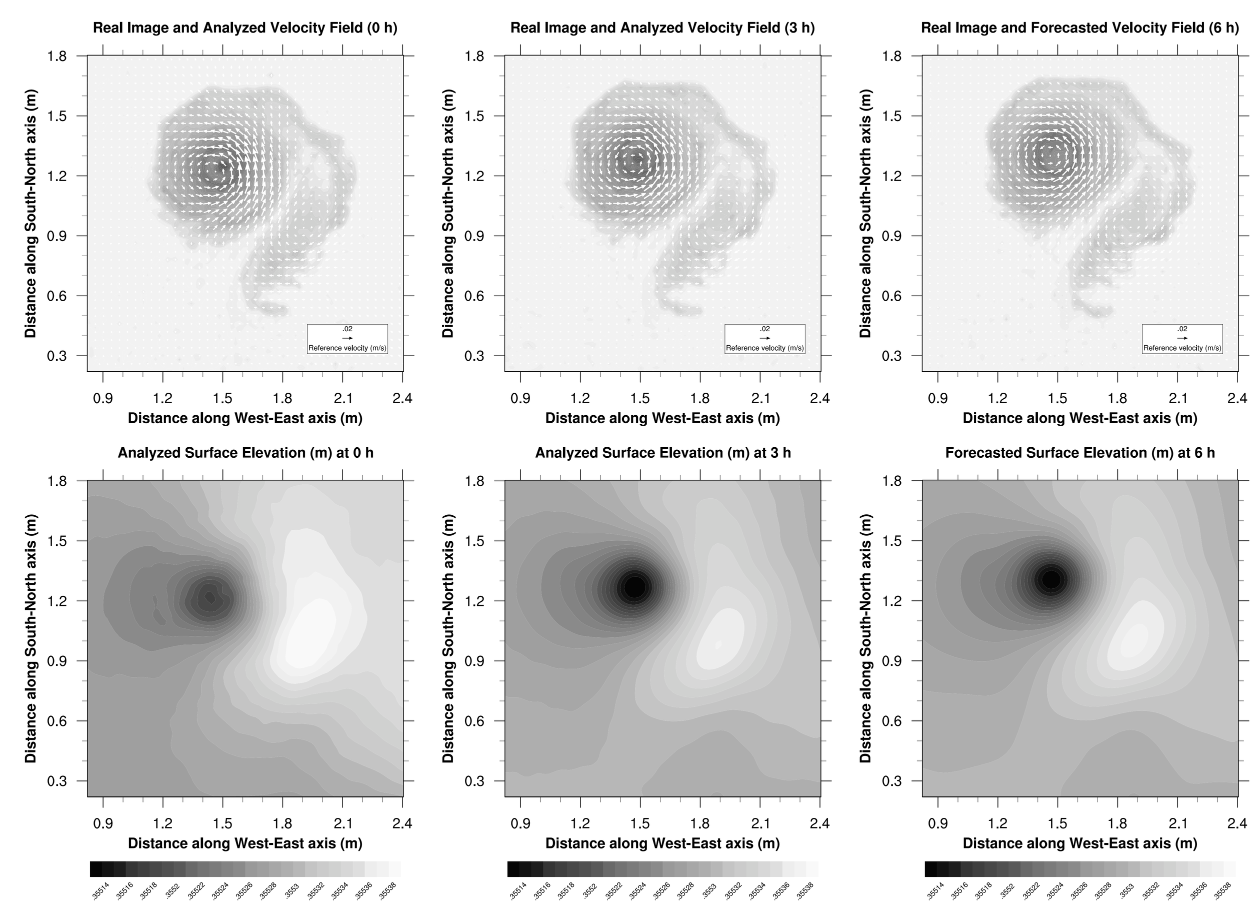
Twin experiments



$t = 24$h. $t = 12$h. $t = 00$h.

Vérité   analyse   True initial velocity   Analysed true velocity   Angular error   RMS error

Real-data experiment



$t = 0$h $t = 3$h $t = 6$h

Conception LaTeX , paquets pgf et TikZ, PyNGL & PyNIO: Innocent Souopgui, Arthur Vidard oct. 2010