Geostrophic turbulence near rapid changes in stratification

Towards High Resolution of Ocean Dynamics and Terrestrial Surface Waters from Space Workshop

Lisbon Oct 2010

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Idea…

- A body of recent work (Klein, Lapeyre and collaborators) suggests that Surface Quasigeostrophic (SQG) dynamics provides at least a partial description of upper-ocean submesoscale flow.
- …but the surface of the ocean is not a rigid boundary, and SQG is, formally, an exotic special case.
- How can we generalize `SQG behavior' to realistic environments? What are the minimum, essential ingredients to get SQG behavior, and what are its limits?
- Claim: SQG turbulence is a generic aspect of geostrophic turbulence near background inhomogenities.

Classic SQG

$z = 0$ rigid lid

$$D_t b = 0 \qquad b = f\partial_z\psi = b_s(x,y,t)$$

$$D_t q = 0 \qquad q = \nabla^2\psi + \partial_z\left(\frac{f^2}{N^2}\partial_z\psi\right) = 0$$

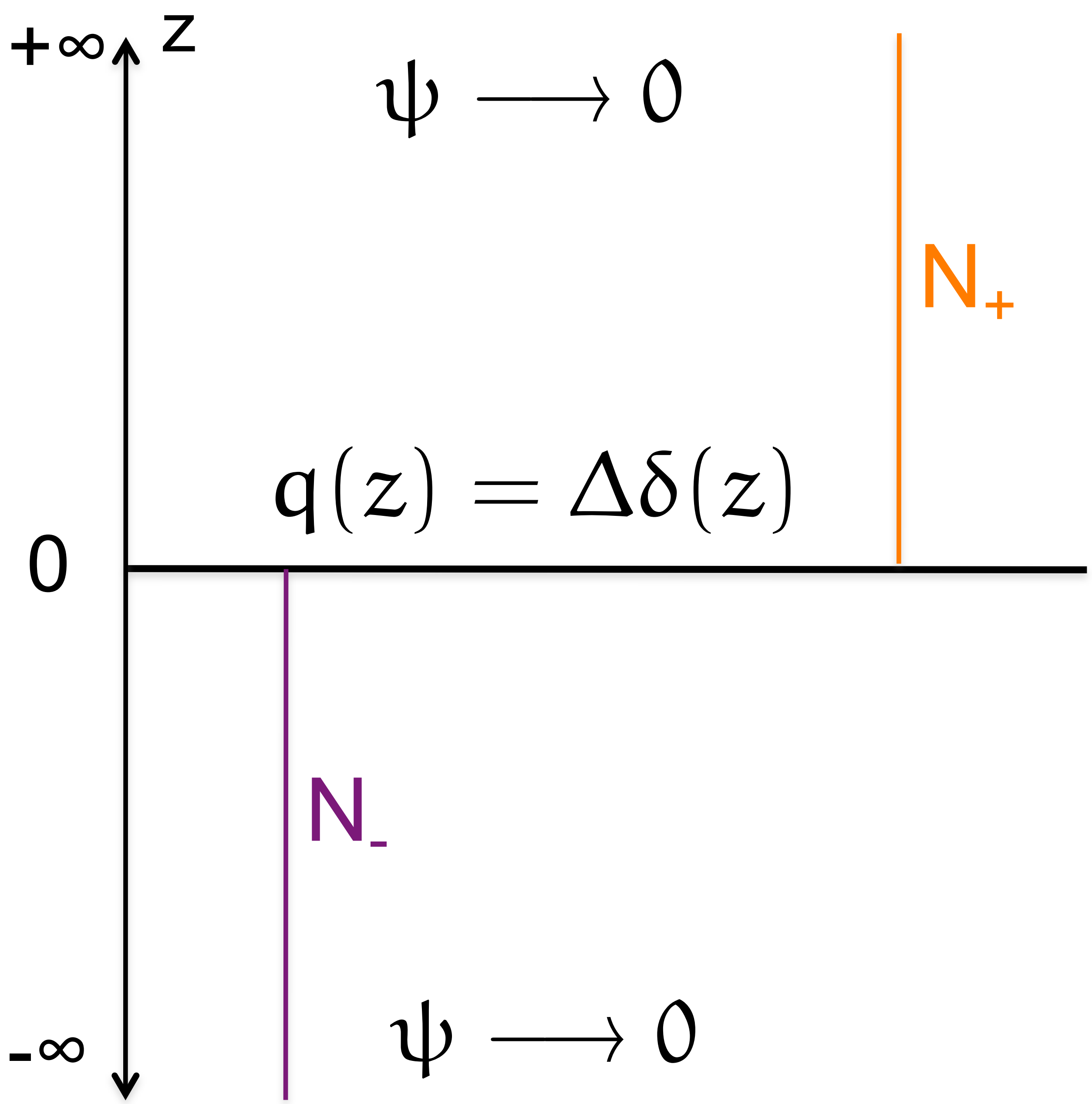
$$z \to -\infty \qquad \psi \to 0$$

Spectral space, constant N => $\hat{\psi}_{\mathbf{K}}(z) = \frac{f}{NK}\exp\left(\frac{NK}{f}z\right)\hat{b}_{\mathbf{K}}(0)$

Kolmogorov scaling => KE spectrum $\propto K^{-5/3}$ in forward b^2 cascade

Blumen (1978), Held et al (1995)

SQG at a discontinuous jump in N



When N has a step-function jump, PV is dominated by delta-fn from dN/dz(z=0):

q advection ➔ b advection

Conservation law at z=0

$$D_t \Delta = 0$$

$$\Delta = \frac{f}{N_+^2} b^+ - \frac{f}{N_-^2} b^-$$

SQG behavior without a surface

Consider buoyancy frequency N(z) with a smooth jump at z=0:

$$
\begin{aligned}
q &= \nabla^2\psi + \partial_z\left(\frac{f^2}{N^2}\partial_z\psi\right) \\
&= \nabla^2\psi + \frac{f^2}{N^2}\partial_{zz}\psi + f\partial_z\left(\frac{1}{N^2}\right)b, \qquad b = f\partial_z\psi
\end{aligned}
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~3D laplacian
when z is rescaled

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Near z=0, conservation of q is like conservation of b, with q above and below relatively small

Green's function for QGPV inversion

Given QGPV operator in spectral space

$$\mathcal{L}\psi(z) \equiv -K^2\psi + \left(\frac{f^2}{N^2}\psi'\right)' = q(z)$$

...we can define a Green's function

$$\mathcal{L}G(z,\xi) = \delta(z-\xi)$$

...such that the streamfunction can be recovered from the PV by integration

$$\psi(z) = \int_{-\infty}^{+\infty} d\xi \; q(\xi)G(z,\xi)$$

Spectrum from GF: Example

Green's function for constant N and z→±∞

$$G(z,\xi) = -\frac{N_0}{2f}\frac{e^{\pm\frac{K}{K_D}(z-\xi)}}{K} \equiv G_0(z,\xi) \qquad K_D = \frac{f}{N_0 H}$$

Consider limits of constant and delta-function PV distributions

$$q(z) = q_0 \quad \Rightarrow \quad \psi_K(z=0) \propto K^{-2}$$

$$q(z) = \Delta\delta(z) \quad \Rightarrow \quad \psi_K(z=0) \propto K^{-1}$$

Spike in q picks out just contribution of G at z=0.

Smooth, but peaked q will do this up to a some small wavenumber proportional to the inverse of the width of the peak

Consider N(z) with smooth jump

Model stratification:

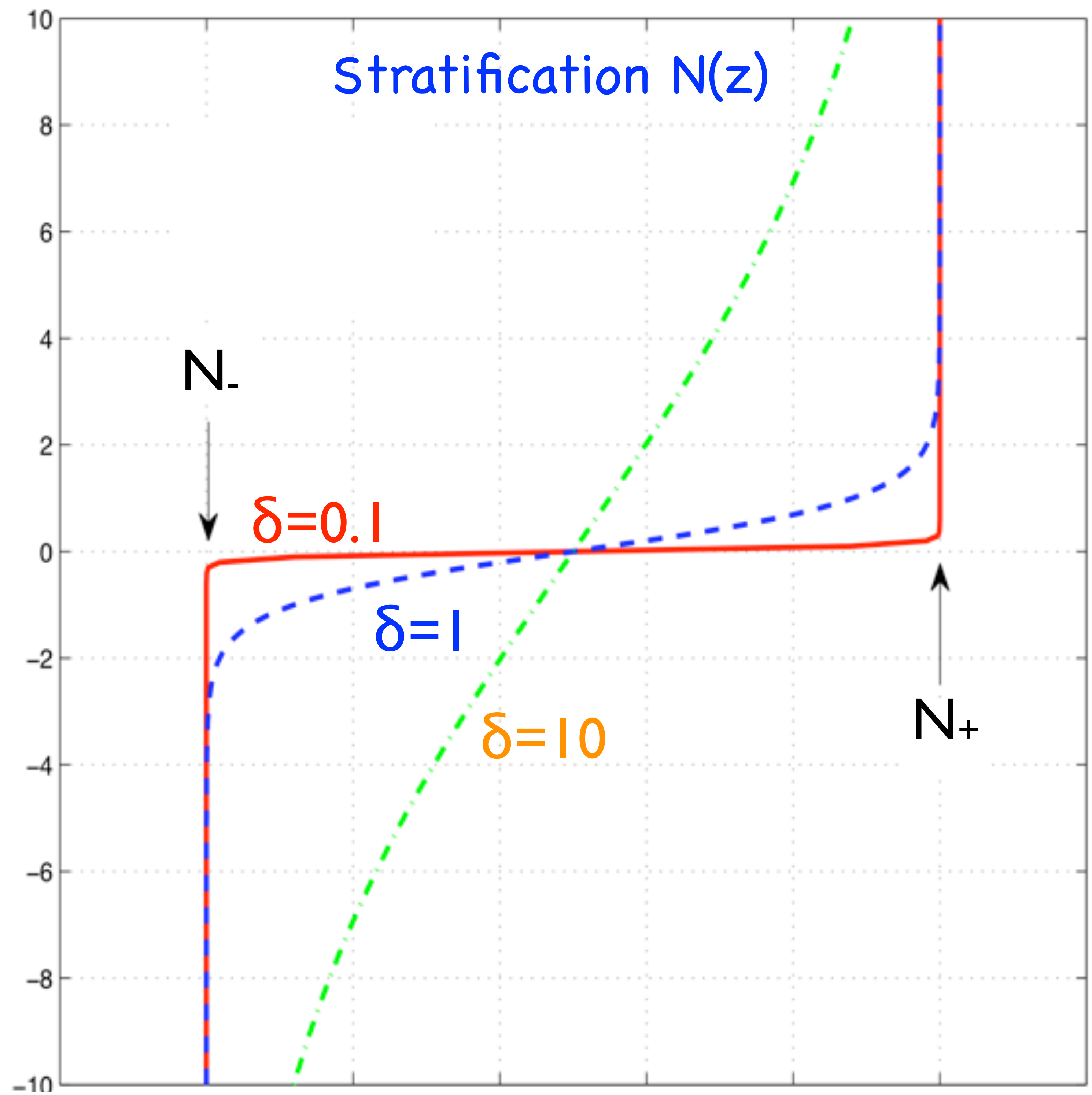
$$N(z) = N_0 + N_d \tanh \frac{z}{\delta}$$

...with

$$N_0 = \frac{N_+ + N_-}{2}$$

$$N_d = \frac{N_+ - N_-}{2}$$

δ is our control parameter



WKB approximation for Green's fn

Consider nondimensional QGPV inversion in horizontally periodic, vertically infinite domain, for each horizontal wavenumber K (we later compute finite-depth G numerically)

$$\mathcal{L}\psi(z) = \epsilon^2\sigma\,\psi'' + \epsilon^2\sigma'\,\psi' - \psi = \frac{q(z)}{K^2}$$

with $\epsilon \equiv \dfrac{f}{N_0 K H}$ and $\sigma(z) \equiv \dfrac{N_0}{N_0 + N_d \tanh z/\delta}$

WKB approximation of Green's function for operator when $\epsilon \ll 1$

$$G(z,\xi) \approx \boxed{-\frac{\sqrt{N(z)N(\xi)}}{2f}\,\frac{e^{\pm\frac{K}{K_D}(z-\xi)}}{K}}\ \boxed{\left[\frac{\cosh z/\delta}{\cosh \xi/\delta}\right]^{\pm\frac{K}{K_\delta}}}$$

...compare to G for N=N_0

$$G_0(z,\xi) = -\frac{N_0}{2f}\,\frac{e^{\pm\frac{K}{K_D}(z-\xi)}}{K}$$

new scale: $\boxed{K_\delta \equiv \dfrac{f}{N_d\,\delta}}$

Relative smoothness of G and q...

$$G(z,\xi) \approx -\frac{\sqrt{N(z)N(\xi)}}{2f}\,\frac{e^{\pm\frac{K}{K_D}(z-\xi)}}{K}\left[\frac{\cosh z/\delta}{\cosh \xi/\delta}\right]^{\pm\frac{K}{K_\delta}}$$

Consider two limiting cases:

1. Peak of PV at z=0: $q(z) \sim \Delta\delta(z)$

$$\psi(z) = \int_{-}^{+} d\xi\; q(\xi)G(z,\xi)$$ $\Rightarrow$ Local SQG relation $\boxed{\psi \sim K^{-1}q}$

2. Large wavenumber: $K \gg K_\delta$

$$G(z,\xi) \sim K^{-2}\delta(z-\xi)$$ $\Rightarrow$ Local 2D relation $\boxed{\psi \sim K^{-2}q}$

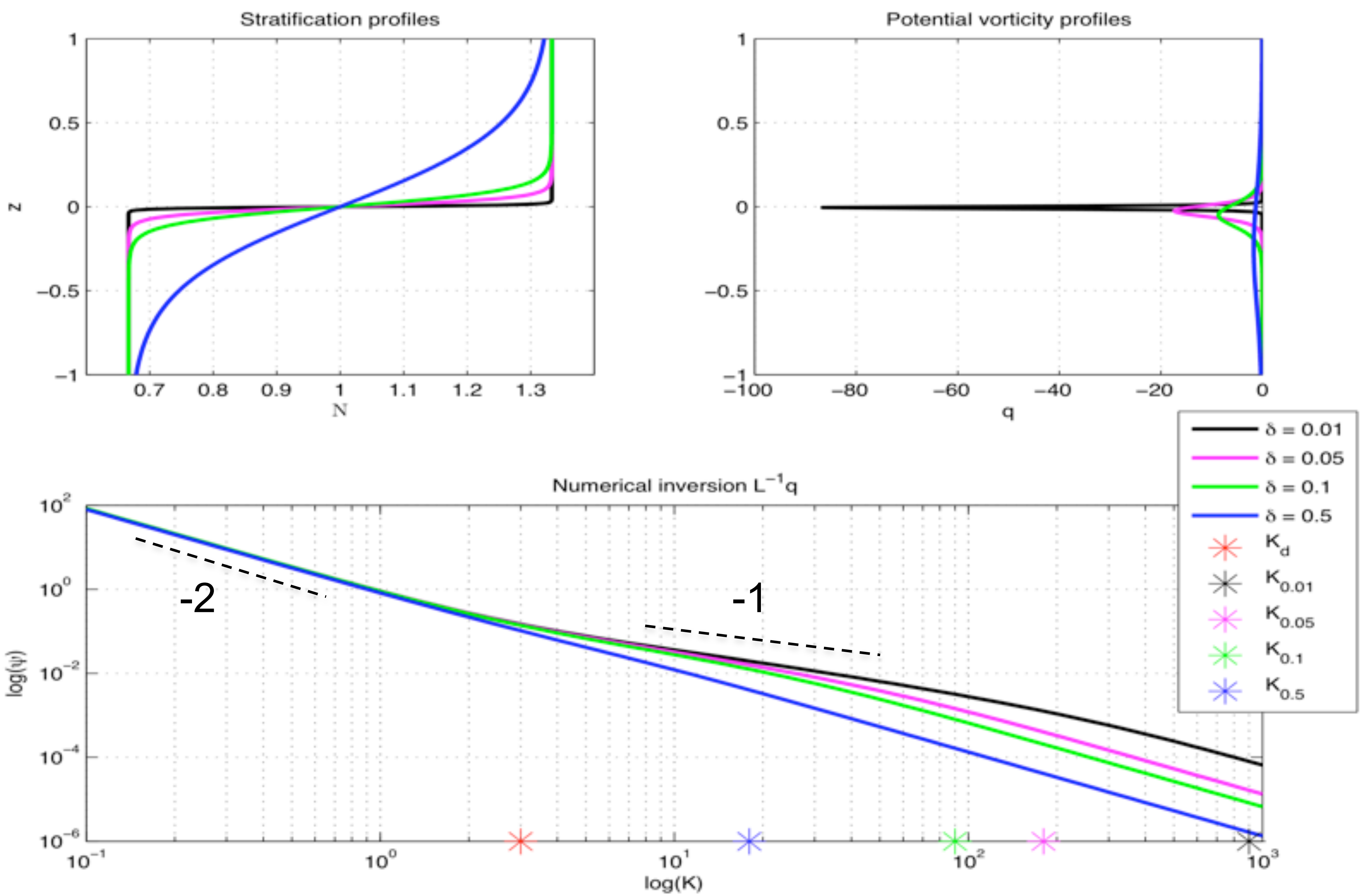
Conjecture

Peaked PV will yield SQG-like dynamics between deformation wavenumber of domain, K_D, and deformation scale of jump, K_δ

Numerical GF spectra

Choose initial PV to be consistent with N

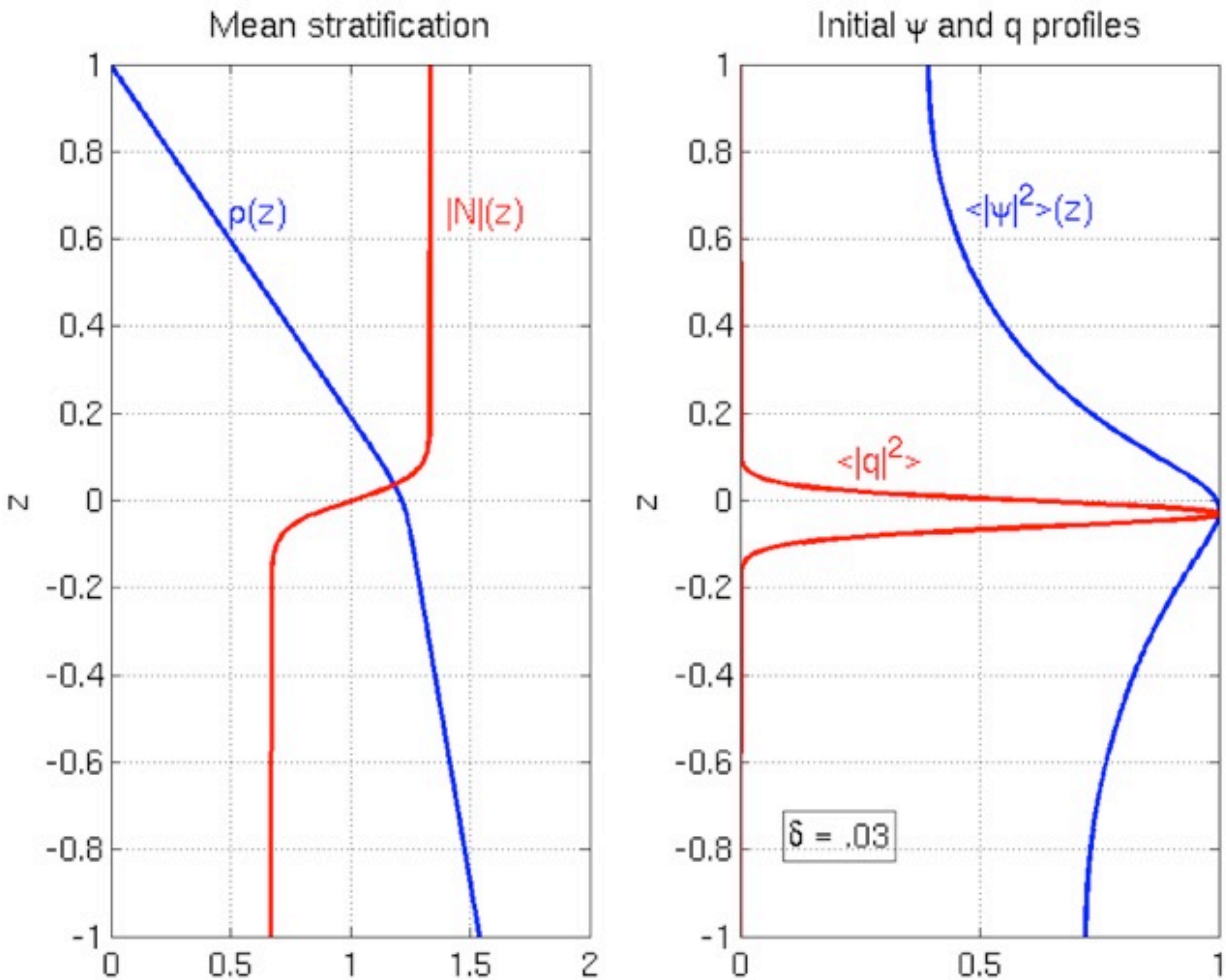
$$q = \frac{f^2}{N^2}\psi'' - 2\frac{f^2 N'}{N^3}\psi' - K^2\psi \quad \text{with} \quad \psi' = 1$$



Spectra of L^{-1} q for G with boundaries at z = +/- 1

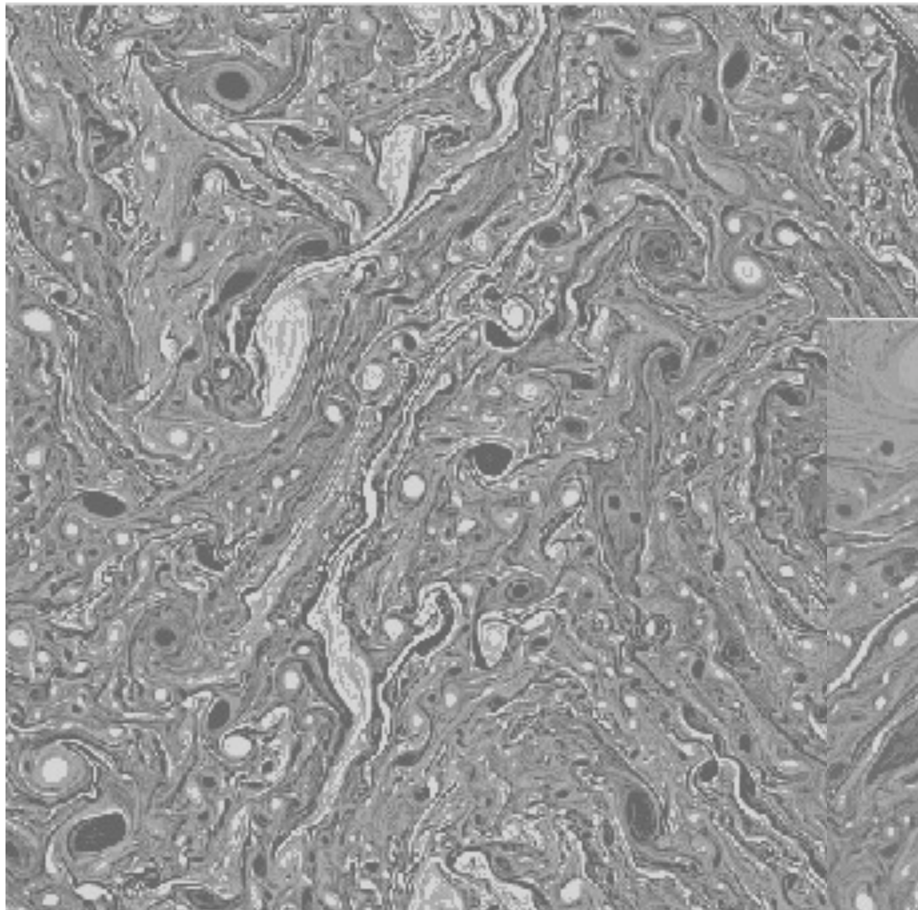
Nonlinear simulations

Freely-evolving, rigid boundaries at z= +/- 1, N(z), initial PV and streamfunction as below. Resolution 512^2 x 400

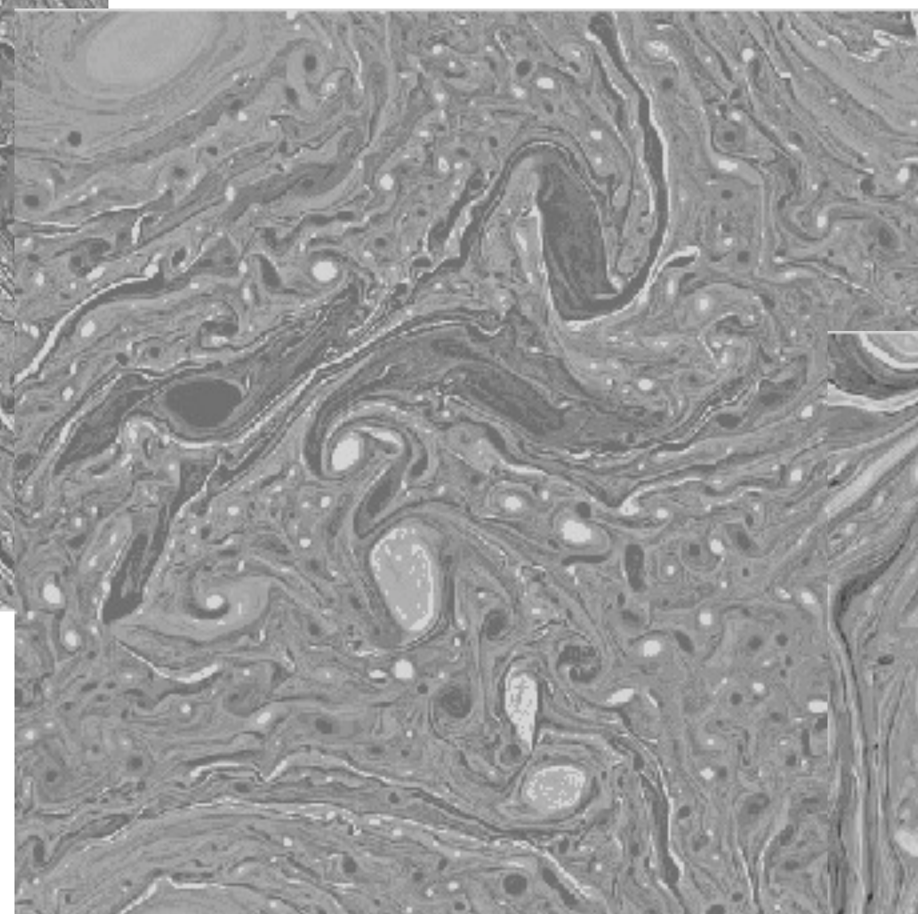


Vorticity at z=0

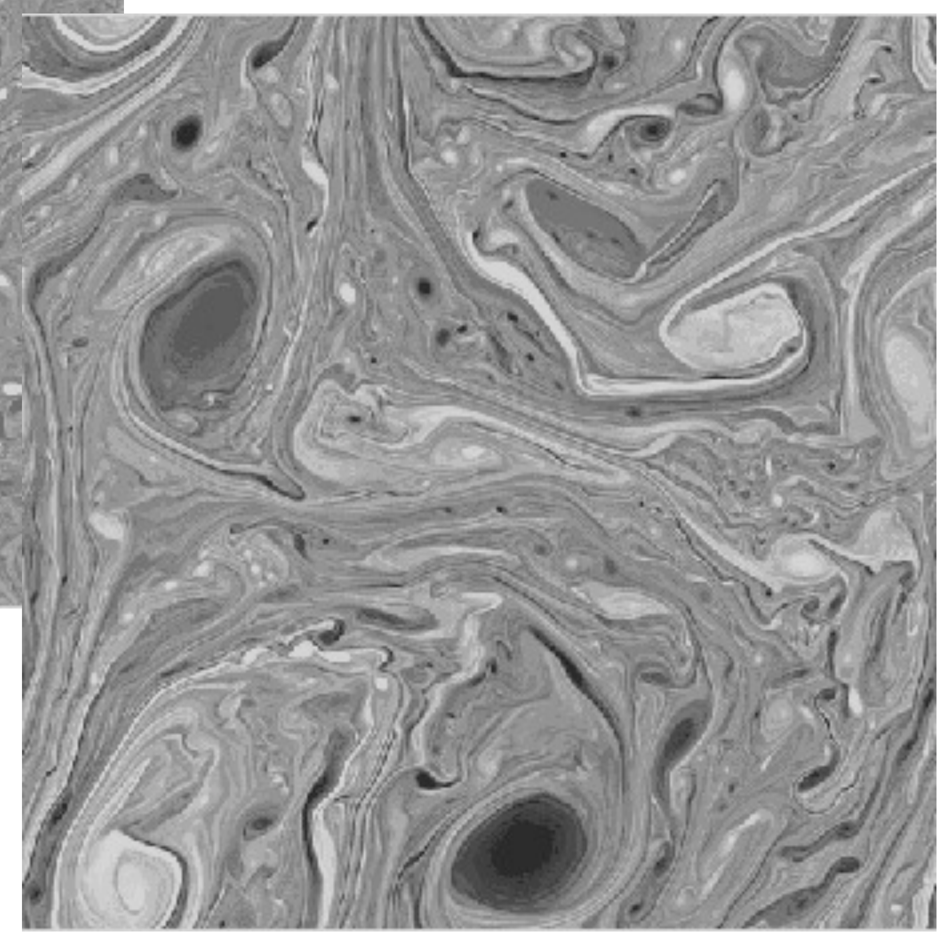
$\delta = 0.01$



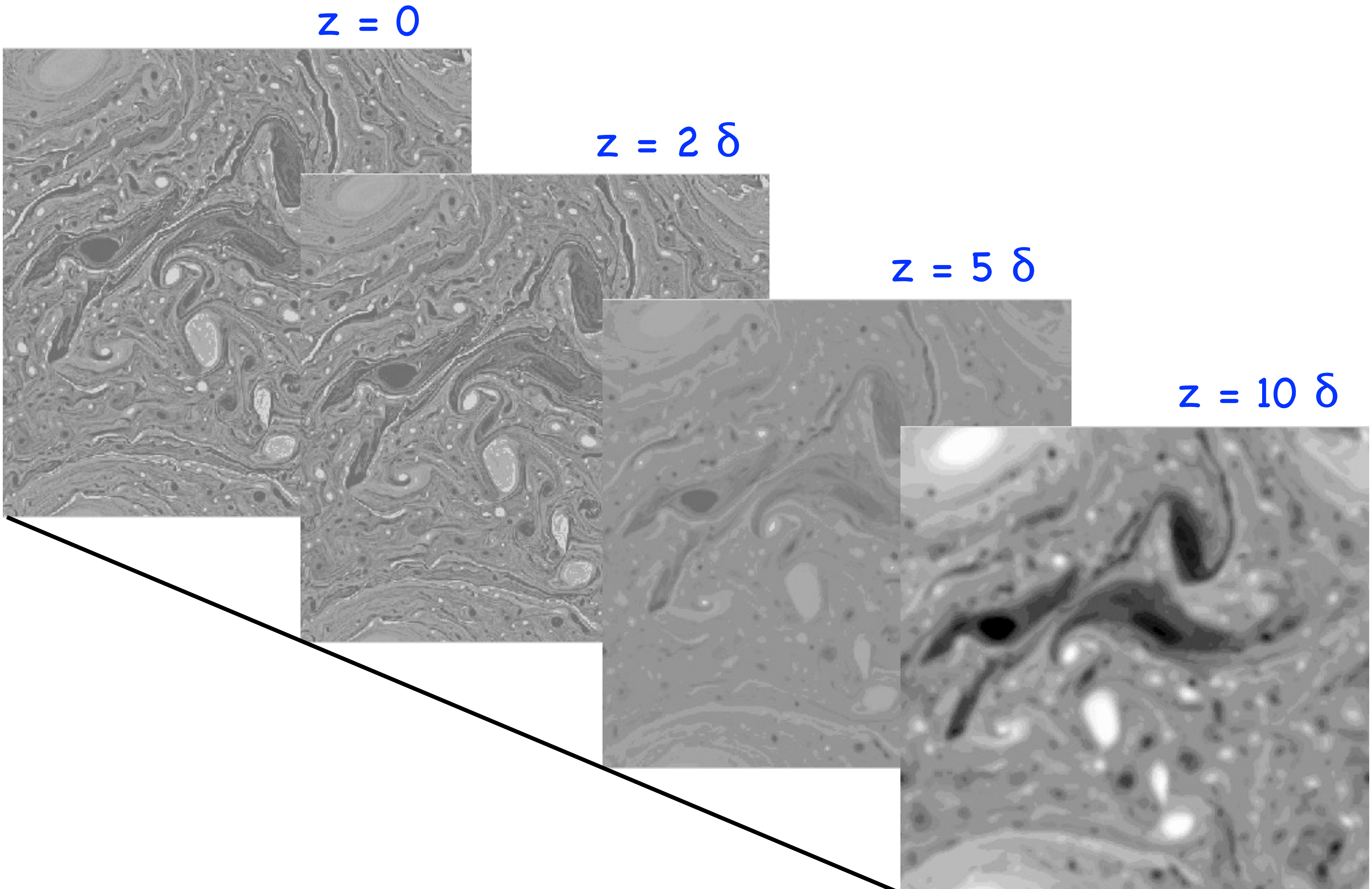
$\delta = 0.03$



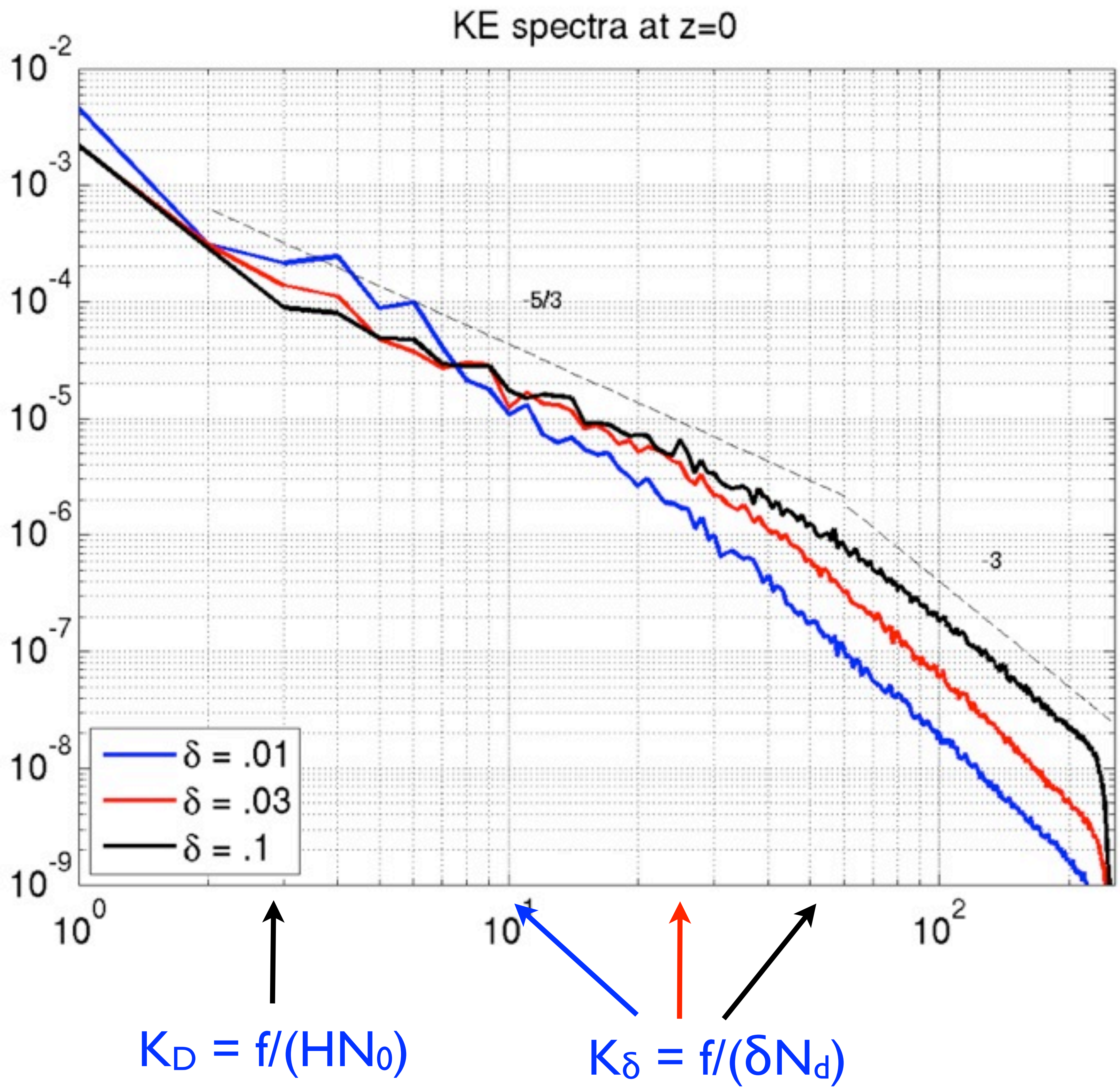
$\delta = 0.1$



Vorticity for $\delta = 0.03$



KE spectra at z=0 for 3 cases



Conclusions

- 'SQG behavior' occurs near vertical changes in stratification N, at wavenumbers K such that $f/(NH) < K < f/(N\delta)$
- The base of the mixed layer is thus the likely generator of observed SQG-like dynamics
- SQG seems to be a generic characteristic of geostrophic turbulence ...
- In real flows (and flows modeled by the full equations), SQG behavior provides a route to loss-of-balance, since the Rossby number increases throughout its forward cascade

Shameless advertisements for other work related to interpreting satellite observations

- Sensitivity of mixing measures to sparse spatial and temporal surface observations (w/ S. Keating - submitted to JPO)
- Projection of QG flow onto vertical modes that efficiently represent both surface and interior dynamics and diagonalize the energy (w/ J. Vanneste)
- Turbulent filtering methods to model unresolved flow (w/ A. Majda and S. Keating)

FTLE for SQG flow with sparse obs