Bayesian Estimation of altimeter echo parameters

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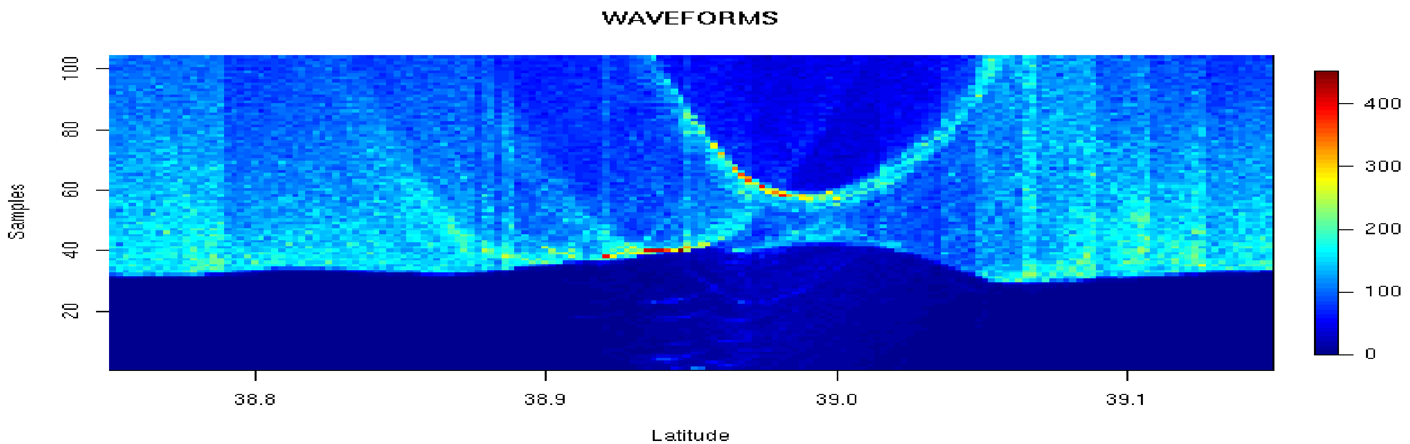
Context of the study

In the current retracking procedures, waveforms are processed independently from the previous ones by comparing the measured altimetric waveform with a return power model (Hayne model) according to least square estimators.



However, consecutive altimetric waveforms are representative of continuous ocean surfaces and **it seems promising** to account from previous WFs when estimating the ocean parameters

- A first step has been done by JPL (E.Rodriguez) when processing the 10 Wfs of the 1s WF-packet at the same time (Topex retracking in RGDR) and solving for 10 ranges but only one SWH and one sigma0 per second
- We have been investigating a **Bayesian approach** combining information coming from the data (contained in the likelihood) and prior knowledge regarding unknown parameters (contained in the parameter prior distribution)

Mathematical formulation

The altimetric signal is modeled using a Brown model

$$y_k = s_k . e_k$$

k=0,…,K-1 ; K nb of WF samples

e_k is a multiplicative speckle noise (*L*: number of integrated pulses) $e_k \sim G(L, L)$

The likelihood of the estimation vector y (the waveform) is :

$$f(y|\boldsymbol{\theta}) = \left[\frac{L^L}{\Gamma(L)}\right]^K \exp\left(-L\sum_{k=1}^{K}\frac{y_k}{s_k}\right)\left(\prod_{k=1}^{K} y_k\right)^{L-1}\left(\prod_{k=1}^{K} s_k\right)^{-L}$$

where θ is the unknown parameter vector (τ, SWH,Pu,…)
y is the observed waveform y=(y_0,…,y_{K-1}) with independant noise samples
f(y/θ) is the product of K Gamma probability density functions

The Maximum likelihood Estimator (MLE) of θ is : $\widehat{\theta}_{\mathrm{MV}} = \arg\max_{\boldsymbol{\theta}} f(y|\boldsymbol{\theta})$

➔ No analytical expression for the Maximum Likelihood Estimator of θ
➔ We currently use a quasi Newton method (approximate solution)

Bayesian estimation : principle

The posterior distribution of θ is defined as :

$$f(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{f(\boldsymbol{y}|\boldsymbol{\theta})f(\boldsymbol{\theta})}{f(\boldsymbol{y})} \propto f(\boldsymbol{y}|\boldsymbol{\theta})f(\boldsymbol{\theta})$$

« A priori » function also called prior distribution

Likelihood function = information coming from the data

where y is the observed waveform and θ is the unknown parameter vector (τ, SWH,Pu, …)

Two possibilities to achieve the estimation of θ

- the minimum mean square error (MMSE) (mean of the posterior distribution)

- the maximum a posteriori (MAP) (method investigated some years ago by E.Rodriguez on Topex data)

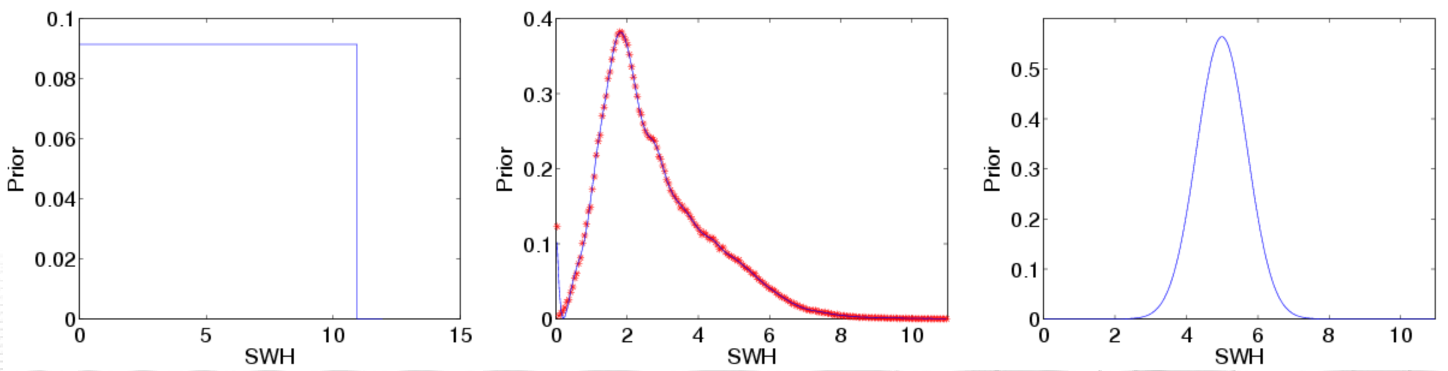
$$\begin{aligned} \widehat{\boldsymbol{\theta}}_{MMSE} &= \mathrm{E}[\boldsymbol{\theta}|\boldsymbol{y}], \\ \widehat{\boldsymbol{\theta}}_{MAP} &= \arg\max_{\boldsymbol{\theta}} f(\boldsymbol{y}|\boldsymbol{\theta})f(\boldsymbol{\theta}) \end{aligned}$$

Prior distributions

Prior distributions summarize the available information regarding the unknown parameters. Different scenarii have been investigated

- Uniform distributions on appropriate intervals,
- Priors computed from parameter estimates along a cycle (fitted by splines),
- Time-varying prior distributions (dynamic distribution=Gaussian distribution centered on the previous estimation).

Example for significant waveheight (SWH)



Posterior distributions

$$f(P_u, \tau, \mathrm{swh}|\mathbf{y}) \propto \exp\left[-L\sum_{k=1}^{K}\frac{y_k}{s_k}\right]\frac{\left(\prod_{k=1}^{K} y_k\right)^{L-1}}{\left(\prod_{k=1}^{K} s_k\right)^{L}} f(P_u)f(\tau)f(\mathrm{swh})$$

- Too complex to derive analytical expression for the MAP and MMSE estimates of θ
- We used Markov Chain Monte Carlo (MCMC) methods which consists in building a Markov Chain θ(t) to generate samples distributed according to the a-posteriori distribution

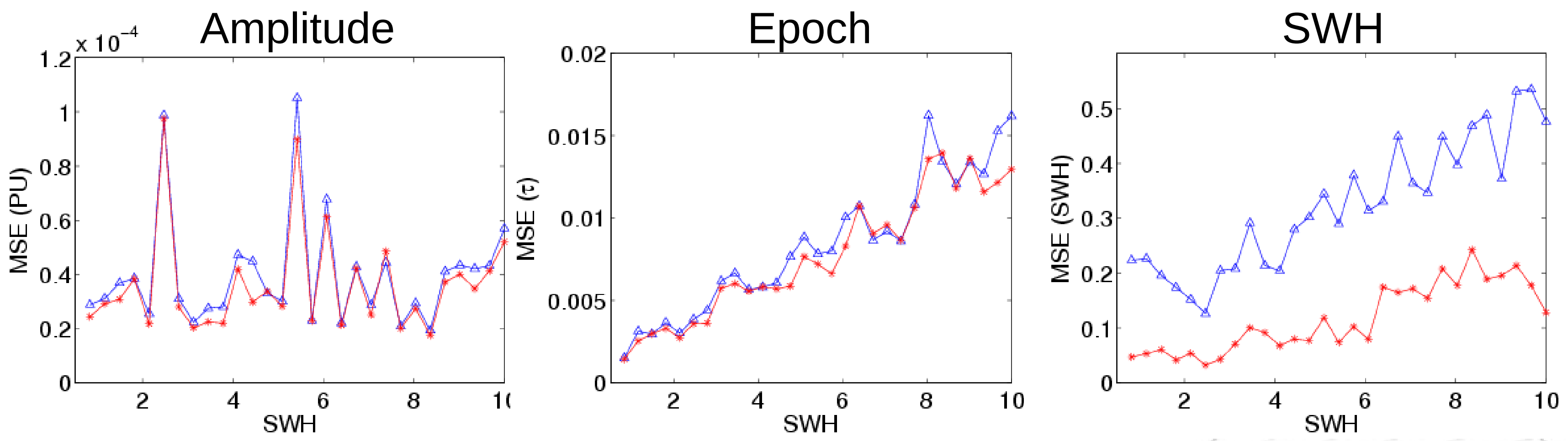
(more details in Severini, Mailhes, Thibaut and Tourneret, IGARSS proceedings, Boston, 2008)

- A hybrid "Metropolis-within-Gibbs" algorithm has been used to generate candidates according a given pdf with an accept/reject procedure.

$$\widehat{\theta}_{\mathrm{MMSE}} \simeq \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\theta}^{(t)}$$

Results on simulation

With uniform priors

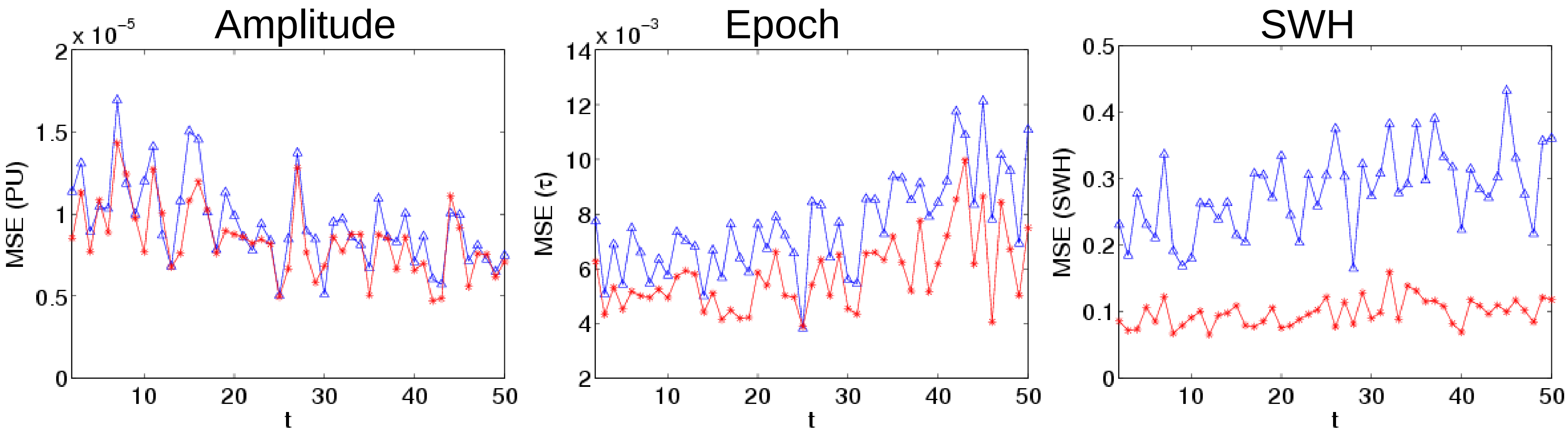


Δ : pseudo MLE, **o** : Bayesian estimator

- Pseudo MLE and Bayes algorithms perform similarly for the parameters Pu and τ
- Better performance for SWH using the Bayesian method.

Results on simulations

With dynamic priors



Δ : pseudo MLE, **o** : Bayesian estimator

Prior distributions at time instant t are defined as Gaussian densities $\mathcal{N}(m; \sigma^2)$, with $m = \theta^{(t-1)}$ and an appropriate variance σ^2,

Performance

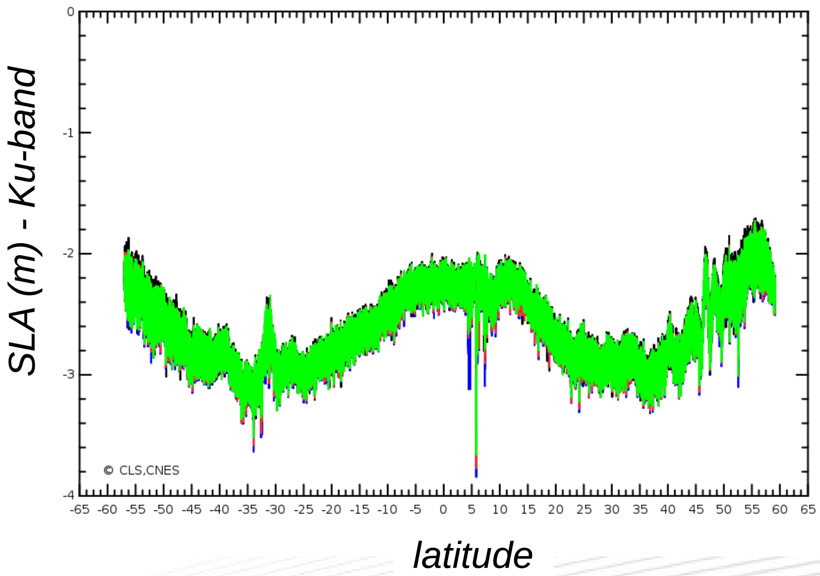
- similar for the estimation of Pu
- Gain for τ
- Gain for SWH

Results on real data

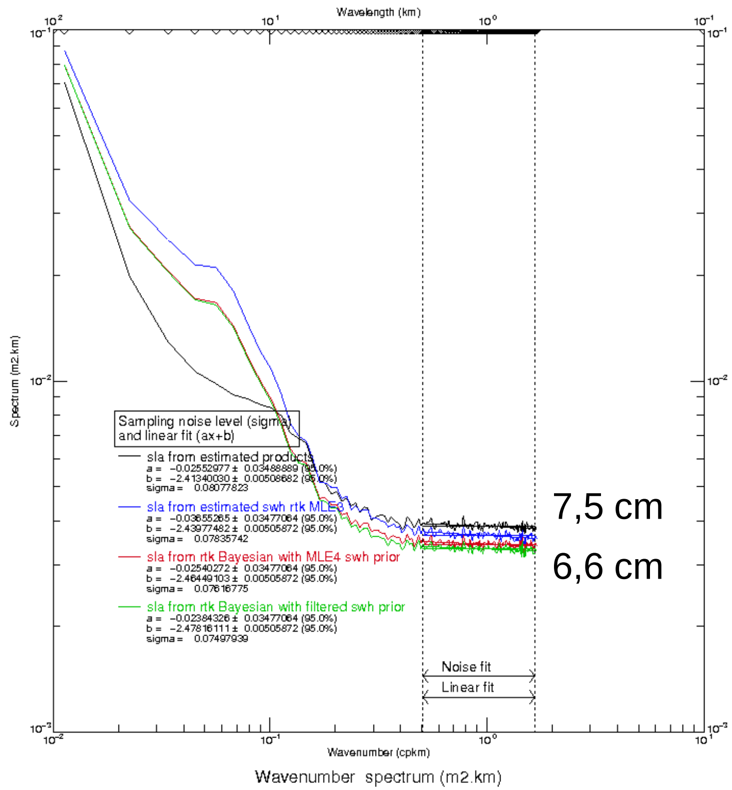
(Jason-2 - tracks 1 to 20 from cycle 16)

Results on SLA

Products
SLA from MLE3
SLA from Bayesian with MLE4 priors
SLA from Bayesian with filtered SWH priors



SLA Power Spectrum (on 20Hz estimates)

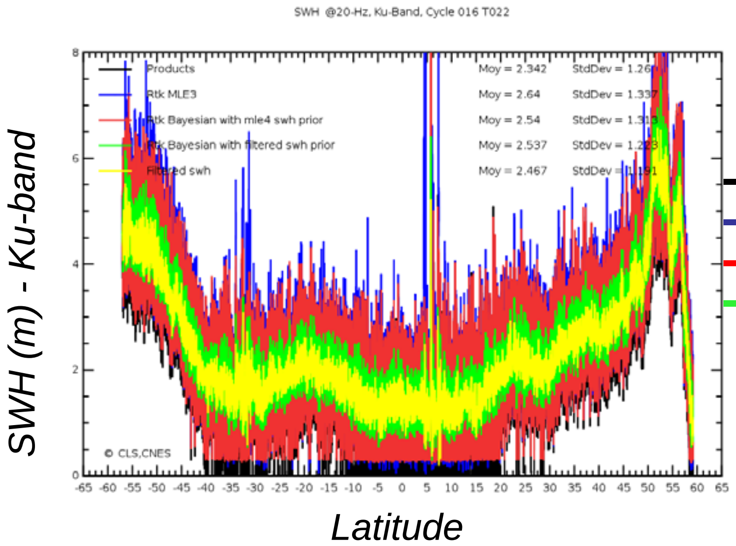


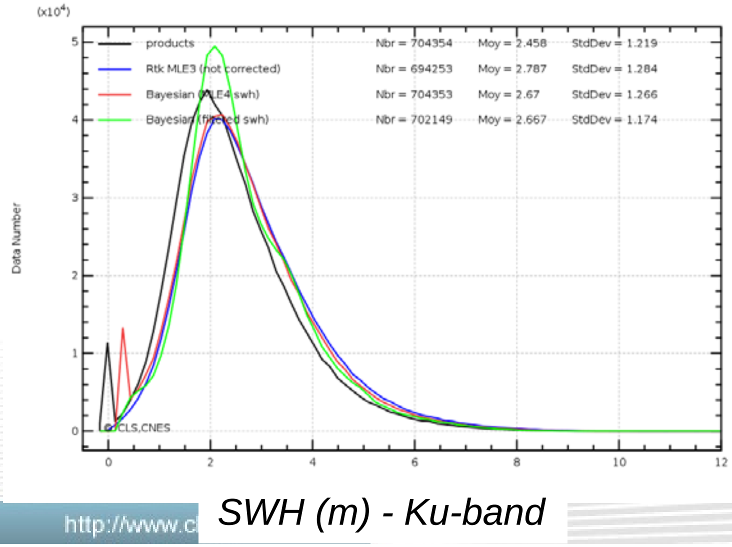
- Small reduction of the SLA noise
- Bayesian depends on priors

Results on real data

(Jason-2 - tracks 1 to 20 from cycle 16)

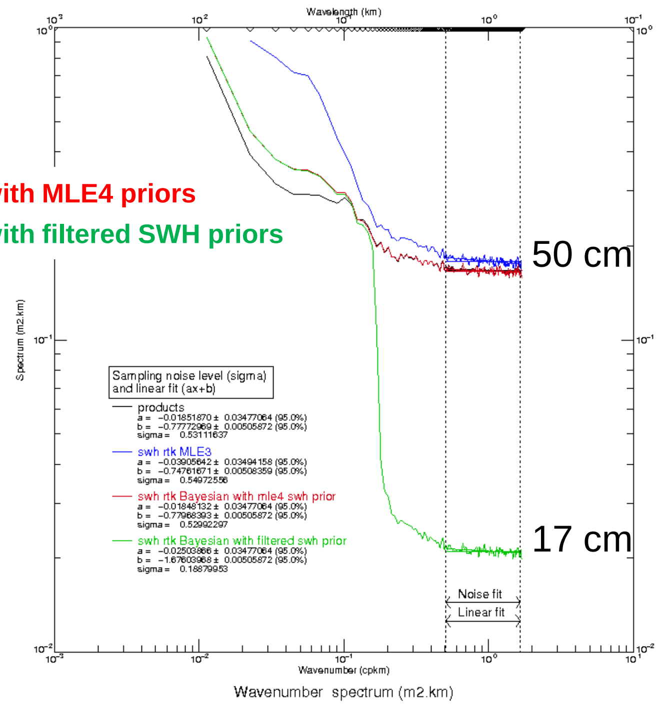
Results on SWH





SWH Power Spectrum (on 20Hz estimates)

- **Products**
- **SWH from MLE3**
- **SWH from Bayesian with MLE4 priors**
- **SWH from Bayesian with filtered SWH priors**

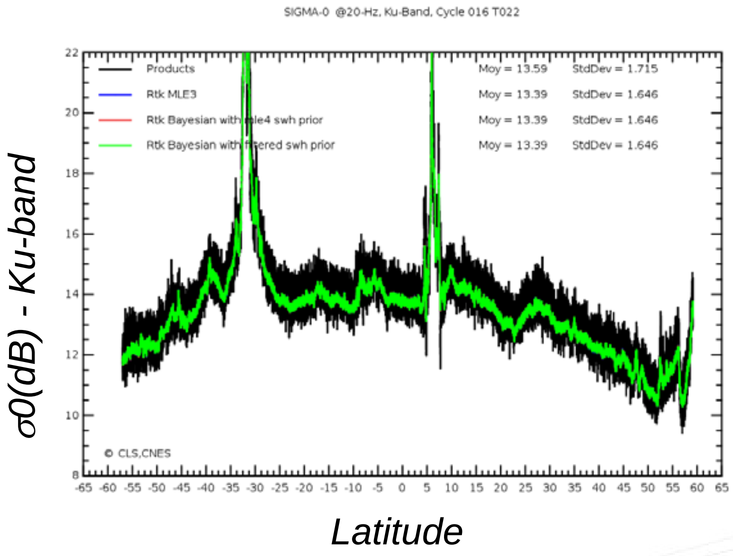


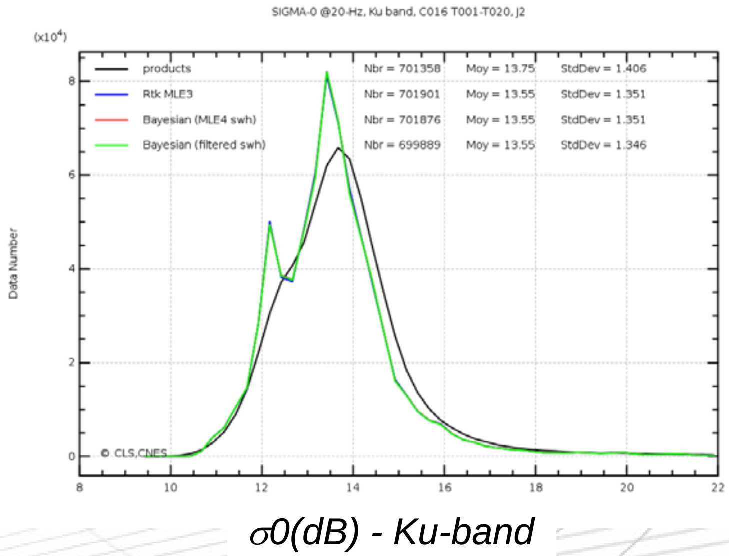
- Strong noise reduction using a filtered prior
- Results linked to the prior choices
- Small biases because products are LUT corrected Bayesian estimates are not corrected

Results on real data

Results on sigma0 (Jason-2 - tracks 1 to 20 from cycle 16)

- **Products**
- **σ0 from MLE3**
- **σ0 from Bayesian with MLE4 priors**
- **σ0 from Bayesian with filtered SWH priors**





- Strong noise reduction on this parameter

Conclusions

- Very promising results
- The method gives the a-posteriori distribution and the confidence interval of the estimations
- However, time consuming method
- This method could be used localy or for very precise estimations (bathymetry) or high rate mean profiles (20Hz)
- Results could be compared with the two passes method developed by NOAA (W.Smith) on typical situations
- Simulations have been done on Median tracking cycles : improvements expected on Diode/DEM cycles. To Be Evaluated …

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