MECHANISMS OF SURFACE INTENSIFICATION BY A MEAN FLOW

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1.INTRODUCTION

Tailleux and McWilliams (2001) previously proposed that the observed intensification of baroclinic Rossby wave speed observed by Chelton and Schlax (1996) could be the result of the surface intensification of the Rossby waves by processes decoupling upper ocean dynamics from bottom ocean dynamics. Processes that could cause such a decoupling are rough topography, nonlinear interactions, or even bottom friction. Such mechanisms do not include a background mean flow. Given the importance of surface intensification proposed by Tailleux and McWilliams (2001), we felt that mean flow effects might be of secondary importance compared to bottom-topographic effects, but the opposite was argued by Killworth and Blundell (2004). Here, we discuss the possibility that surface intensification be entirely due to the background mean flow, rendering the bottom boundary condition ineffective.

2. SCHRODINGER EQUATION FOR LINEAR ROSSBY WAVES

Following Chelton & Schlax (1996), mean flow and topographic effects are accounted for as part as a generalized eigenvalue problem. This can be done from linearized quasi-geostrophic potential vorticity equation, as shown in Tailleux and Maharaj (2004), correcting the previous approach by Killworth and Blundell (2004) (see companion poster). The eigenvalue problem corresponding to the pressure perturbation is given by:

$$(\bar{u}-c)\left(\frac{d}{dz}\left(\frac{f_0^2}{N^2}\frac{dF}{dz}\right)-K^2F\right)+\left(\beta-\frac{d}{dz}\left(\frac{f_0^2}{N^2}\frac{d\bar{u}}{dz}\right)\right)F=0$$

$$(\bar{u}-c)\frac{dF}{dz}=\left(\frac{d\bar{u}}{dz}+\frac{N^2c}{g}\right)F, \qquad z=0$$

$$(\bar{u}_b-c)\frac{dF}{dz}=\left(\frac{d\bar{u}}{dz}+\frac{N_b^2}{f_0}\left(\frac{\partial H}{\partial y}-\frac{k_y}{k_x}\frac{\partial H}{\partial x}\right)\right)F, \qquad z=-H(x,y)$$

It is possible to show that the transformation F = N G/f yields the following classical Schrodinger equation for the function G:

$$\frac{d^2G}{dz^2}+V(z,c,K,\bar{u},N)G=0$$

Where V is the Schrodinger potential, u is the background zonal mean flow, c is the zonal phase speed, kx and ky the zonal and meridional wavenumbers, N the buoyancy frequency, g the gravitational acceleration, and H the total ocean depth.

3. ANALYSIS OF THE POTENTIAL

After some manipulation, one shows that the potential can be expressed as follows:

$$V=\frac{1}{\bar{u}-c}\left\{\frac{\beta N^2}{f_0^2}-\frac{d^2\bar{u}}{dz^2}+\frac{2}{N}\frac{d\bar{u}}{dz}\frac{dN}{dz}\right\}-\frac{N^2K^2}{f_0^2}-N\frac{d^2}{dz^2}\frac{1}{N}$$

For the problem to remain physical, it is necessary to avoid a critical layer, i.e., one needs u-c>0 over the whole ocean depth. If so, one sees that the first term within brackets contributes to make the potential positive, and hence the solution oscillatory in the vertical. One sees, however, that vertical shear can sometimes oppose the first term to make it negative over some region in the vertical, creating a turning point, and the possibility of surface wave trapping. The other terms demonstrate that dispersive effects due to finite wavenumbers can also contribute to make the potential negative, as well as the vertical variations in the buoyancy frequency.

4.RAY APPROACH

In order to investigate the possibility of surface wave trapping, a ray tracing approach was developed for the long wave equations, by using the following profiles for the zonal velocity and buoyancy profiles:

$$U(z)=U_s e^{\frac{z}{2Hh_s}}\cos(k\pi z)$$

$$\frac{N(z)}{f}=\frac{N}{f_{\min}}+\left(\frac{N}{f_{\max}}-\frac{N}{f_{\min}}\right)\left(\frac{z-H}{H}\right)^2$$

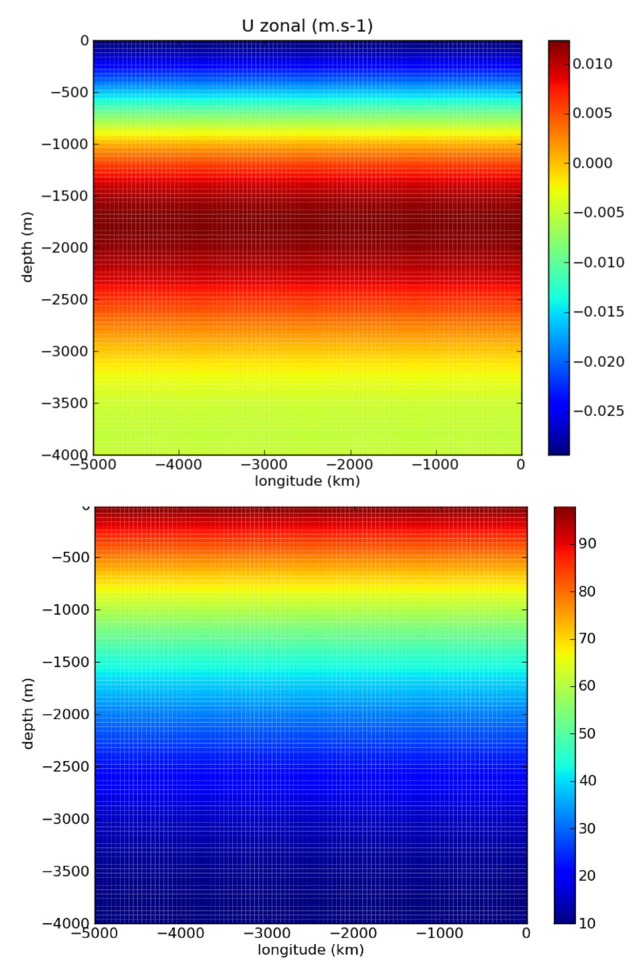

Fig. 1: *longitude/vertical section of the zonal profile used in the ray calculation (top panel). Longitude/vertical section of the buoyancy profile (bottom panel).*

The resulting dispersion relation, and ray equations are given by the following expressions, with the result illustrated in Fig. 2:

$$\omega=Uk-\frac{k}{k^2+(fm/N)^2}\left[\beta-\frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial U}{\partial z}\right)\right]$$

$$\frac{dk}{dt}=-\frac{\partial\Omega}{\partial x}, \quad \frac{dm}{dt}=-\frac{\partial\Omega}{\partial z} \qquad \frac{dx}{dt}=\frac{\partial\Omega}{\partial k}, \quad \frac{dz}{dt}=\frac{\partial\Omega}{\partial m}$$

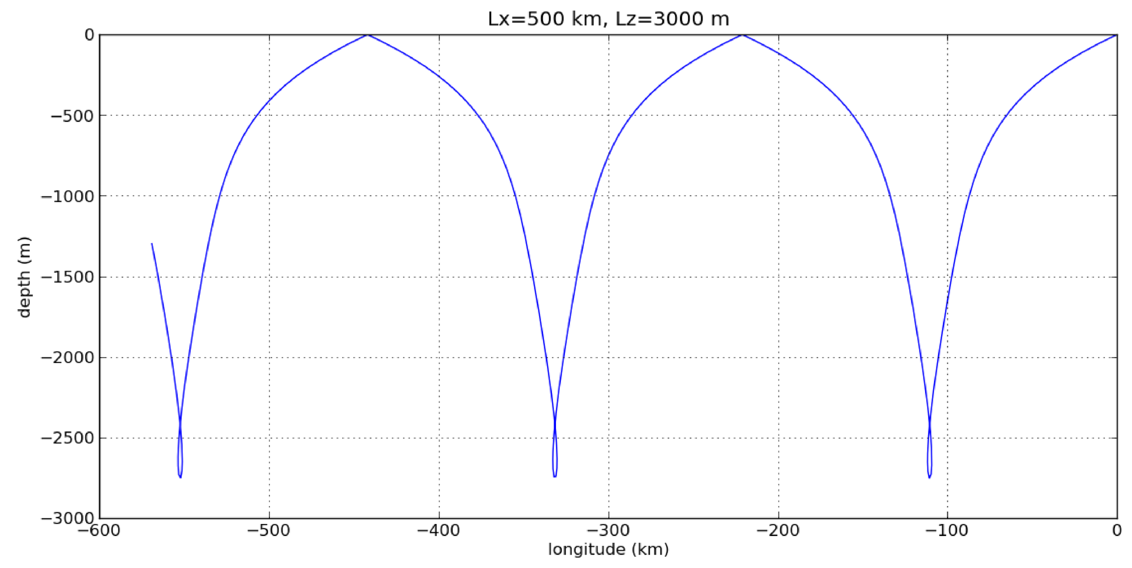

Fig. 2: *Rays corresponding to the above velocity and buoyancy profiles. Such rays display surface-trapping of the energy.*

FUTURE WORK:

These preliminary results suggest that a background mean flow may be such as to permit the surface trapping of the Rossby wave energy. Future work will seek to establish the general conditions under which this happens, and whether the surface trapping is exacerbated by finite wavenumbers, as is suggested by the formulation in terms of a Schrodinger equation.

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5. CONCLUSIONS The generalized eigenvalue problem accounting for a background mean flow and topography previously developed by Killworth and Blundell (2004), and corrected by Tailleux and Maharaj (2010) can be transformed into a Schrodinger equation, which is the form suitable to investigate the possibility for a background mean flow to trap Rossby wave energy near the surface. Preliminary results based on a ray approach suggests the possibility of such an effect, which further work will try to confirm.