Semi-analytical models for delay/Doppler altimetry

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OSTST 2013, Boulder, USA

Summary

- Context / Introduction
- Semi-analytical model (3 parameters)
- Semi-analytical model (5 parameters)
- Conclusions & perspectives

Context

Various solutions to retrack altimeter waveforms

Model	Analytical solution (model + derivatives)	Analytical solution (model without derivatives)	Semi-Analytical solution (double convolution)	Simulated numerical solution (double convolution)
Retracking	Newton-Raphson iterative solution	Levenberg-Marquardt solution		Newton-Raphson iterative solution Echos Data Base

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BROWN, Current LRM processing

Context

Various solutions to retrack altimeter waveforms

Model

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- Analytical solution (model without derivatives)
- Semi-Analytical solution (double convolution)
- Simulated numerical solution (double convolution)

Retracking

- Newton-Raphson iterative solution
- Levenberg-Marquardt solution
- Newton-Raphson iterative solution (Echos Data Base)

SAMOSA SAR solution

Université de Toulouse

Context

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HALIMI SAR solution

Context

Various solutions to retrack altimeter waveforms

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Analytical solution (model + derivatives)

Analytical solution (model without derivatives)

Semi-Analytical solution (double convolution)

Simulated numerical solution (double convolution)

Retracking

Newton-Raphson iterative solution

Levenberg-Marquardt solution

Newton-Raphson iterative solution

Echos Data Base

CNES CPP SAR solution

Introduction

- 3 parameter study
  - Description of the semi-analytical model
  - Parameter estimation
- 5 parameter study
  - Description of the semi-analytical model
  - Parameter estimation

Summary

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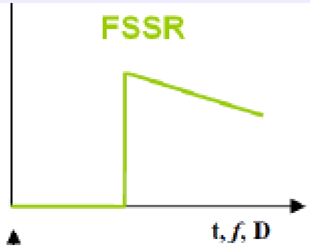
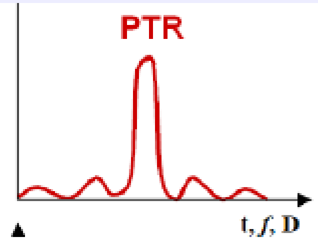
Semi-analytical models for delay/Doppler altimetry

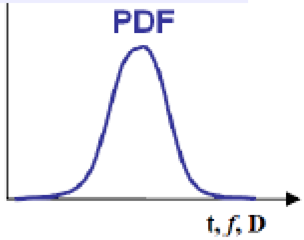
3 parameters (DDA3)

Conventional altimetry

$$\boldsymbol{P}(t) = \text{FSSR}(t) \otimes \text{PTR}(t) \otimes \text{PDF}(t)$$



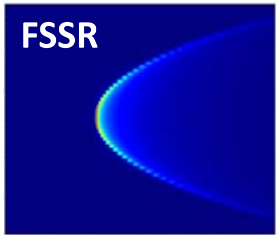


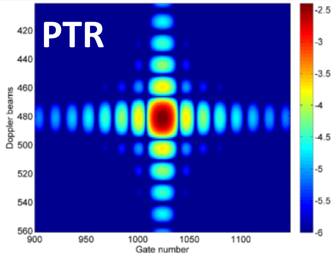


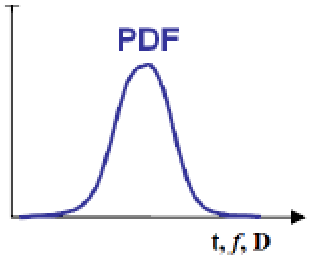
Doppler altimetry [2] [3]

$$\boldsymbol{P}(t,n) = \text{FSSR}(t,n) \otimes \text{PTR}(t,n) \otimes \text{PDF}(t)$$

- n : Doppler band or frequency







Semi-analytical models for delay/Doppler altimetry

3 parameters (DDA3)

$$P(t,f) = \mathrm{FSIR}(t,f) * \mathrm{PDF}(t) * \mathrm{PTR}(t,f)$$

$$\mathrm{FSIR}(t_k,n) = P_u \exp\left[-\frac{4c}{\gamma h} t_k\right]\left[\frac{\phi_{k,n+1}(t_k) - \phi_{k,n}(t_k)}{\pi}\right] U(t_k)$$

- $t_k = kT - \tau_s$
- $k = 1, \cdots, KN_t$ with $K = 128$
- $n = 1, \cdots, NN_f$ with $N = 64$ beams
- $\phi_{k,n} = \mathrm{Re}\left[\arctan\left(\frac{y_n}{\sqrt{\rho^2(t_k) - y_n^2}}\right)\right]$
- $y_n = \frac{h\lambda}{2v_s} f_n$
- $\rho(t_k) = \sqrt{hct_k}$

A. Halimi et al. ``A semi-analytical model for delay/Doppler altimetry and its estimation algorithm '', IEEE TGRS 2013, to appear.

Semi-analytical models for delay/Doppler altimetry

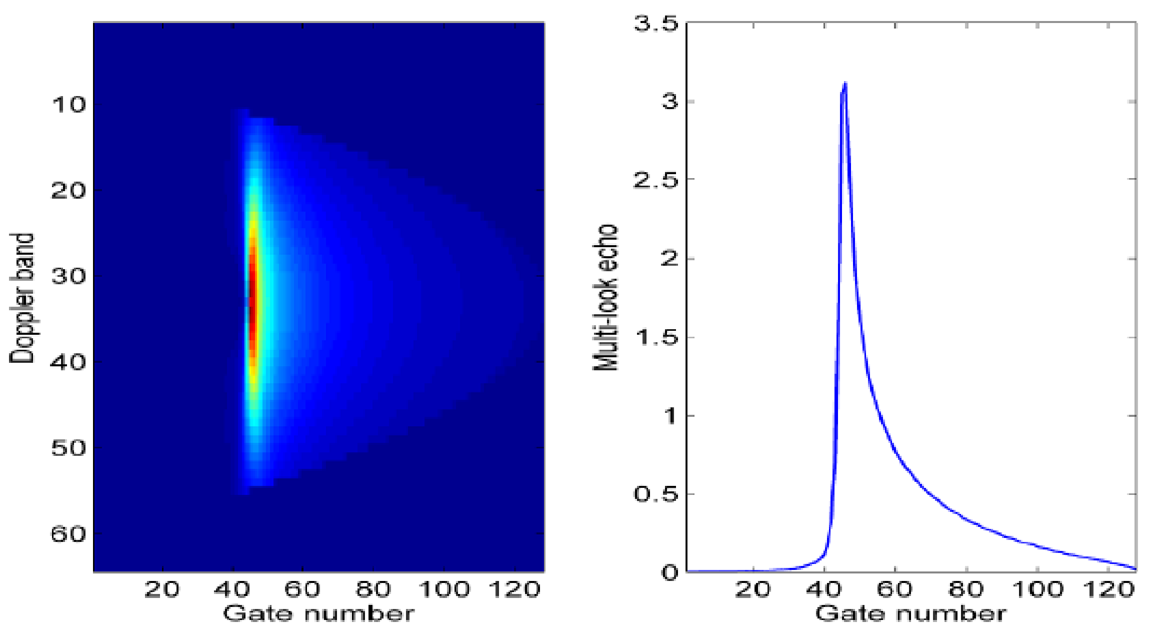
3 parameters (DDA3)

Multi-look echo

$$P(t,n) = \mathrm{FSSR}(t,n) \otimes \mathrm{PTR}(t,n) \otimes \mathrm{PDF}(t)$$

$$s(t) = \sum_{n=1}^{64} P\left[t - \delta t(n), n\right]$$

Geometrical migration correction



Parameter Estimation

- Least squares estimation

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \boldsymbol{F}(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \frac{1}{2}\sum_{k=1}^{K}\left[y_k - s_k(\boldsymbol{\theta})\right]^2$$

  - $\boldsymbol{y} = (y_1, \ldots, y_K)^T$ is the observed multi-look echo
  - $\boldsymbol{s} = (s_1, \ldots, s_K)^T$ is the proposed model
  - $\boldsymbol{\theta} = (P_u, \mathrm{SWH}, \tau_s)^T$ is the parameter vector to estimate

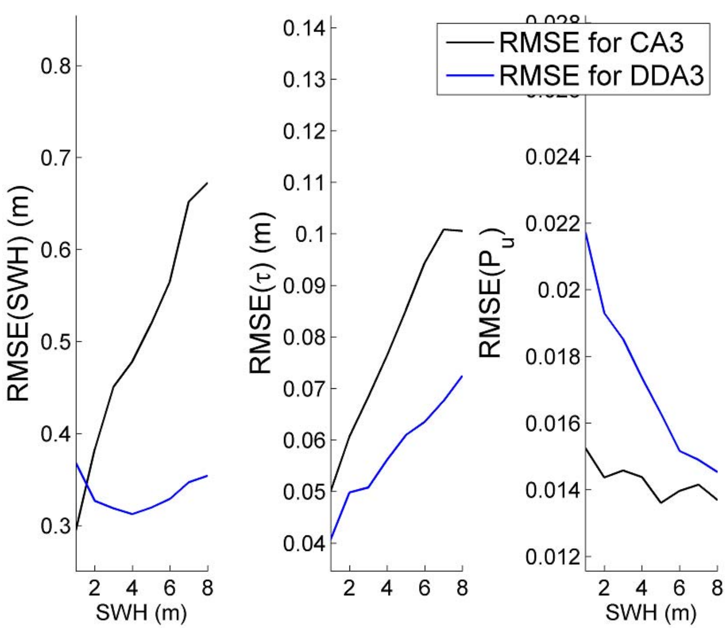
- Levenberg-Marquardt algorithm

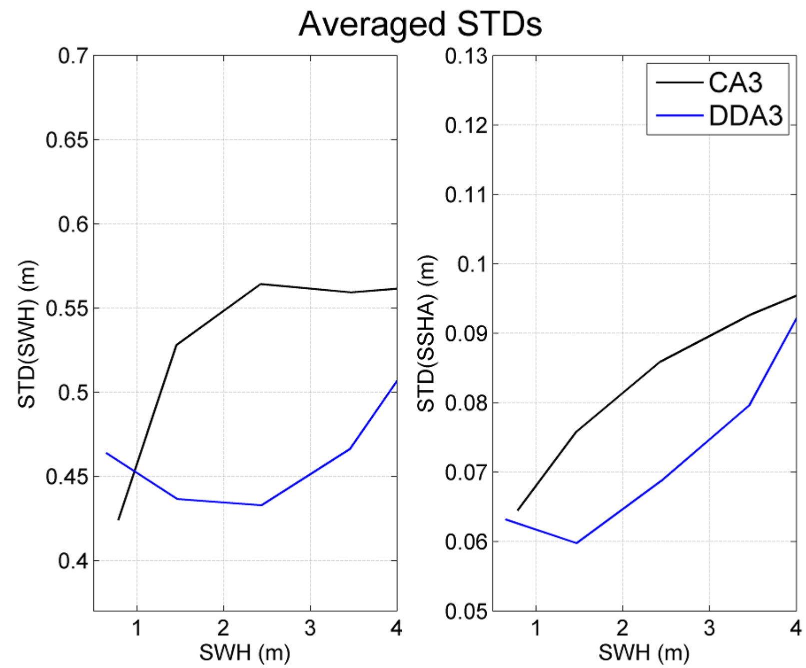
Semi-analytical models for delay/Doppler altimetry

Simulation Results

Already presented at OSTST 2012

Synthetic data	Real data





Improvement factor at SWH = 2 m

Synthetic data	Real data
1.19 for SWH	1.28 for SWH
1.24 for τ	1.26 for τ

Summary

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Semi-analytical models for delay/Doppler altimetry

5 parameters (DDA5)

$$\mathrm{FSIR}(k,n) = \frac{P_u}{2\pi}\left(1+\frac{ct_k}{2h}\right)^{-3} U(t_k)$$

$$\times \left\{ \int_{\phi_{k,n}}^{\phi_{k,n+1}} \exp\left\{ f\left[\tilde{\phi}-\phi, \epsilon(k), \xi\right]\right\} d\phi + \int_{\phi'_{k,n}}^{\phi'_{k,n+1}} \exp\left\{ f\left[\tilde{\phi}-\phi, \epsilon(k), \xi\right]\right\} d\phi \right\}$$

with

$$f\left[\tilde{\phi}-\phi, \epsilon(k), \xi\right] = -\frac{4}{\gamma}\left[1-\frac{\cos^2(\xi)}{1+\epsilon^2(k)}\right] + b(k,\xi)$$
$$+ \; a(k,\xi)\cos\left(\tilde{\phi}-\phi\right) - b(k,\xi)\sin^2\left(\tilde{\phi}-\phi\right)$$

- ξ and $\tilde{\phi}$ represents the antenna mispointing angles
- $a(k,\xi) = \frac{4\epsilon(k)}{\gamma}\frac{\sin(2\xi)}{1+\epsilon^2(k)}$ and $b(k,\xi) = \frac{4\epsilon^2(k)}{\gamma}\frac{\sin^2(\xi)}{1+\epsilon^2(k)}$.

- ξ and $\tilde{\phi}$ are the mispointing angles (related to ξ_{ac} and ξ_{al})

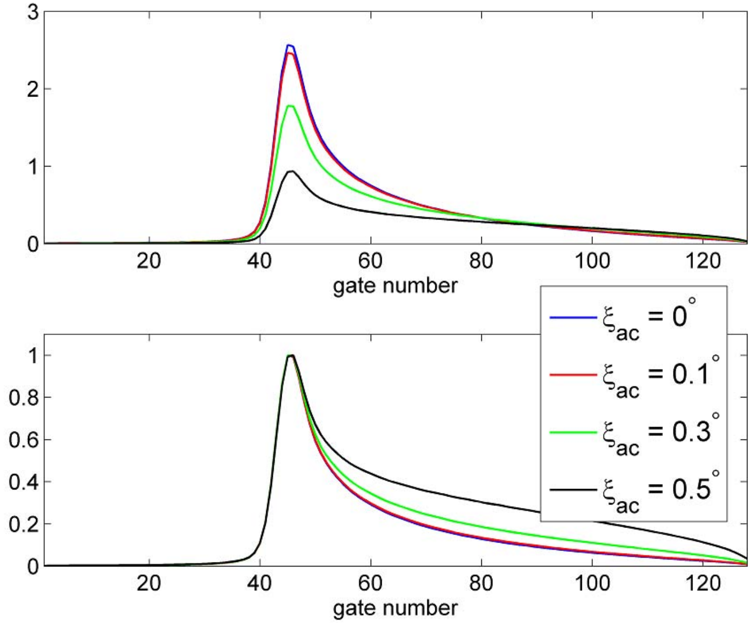
Semi-analytical models for delay/Doppler altimetry

5 parameters (DDA5)

- Effects of across-track and along-track mispointing

Already observed by Gommenginger, OSTST - 2011

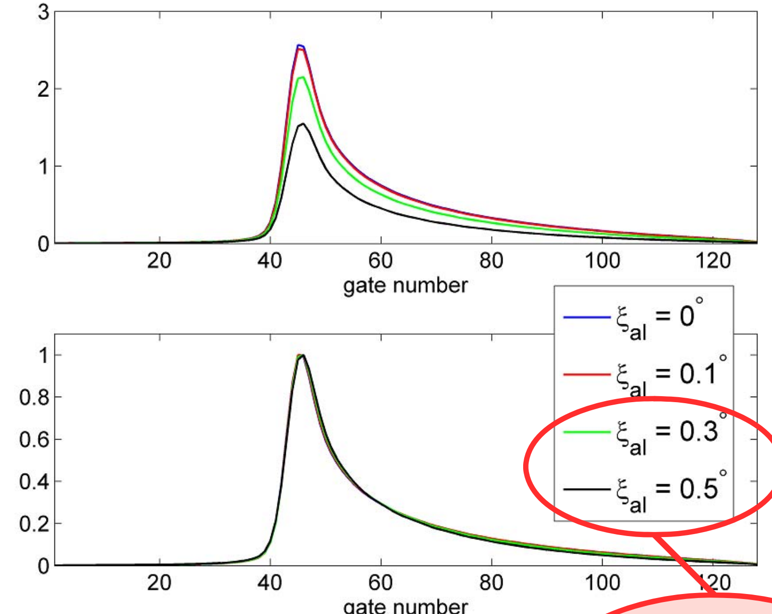
Effect of ξ_{ac}



Changes the Shape and the Amplitude

Normalized echos

Effect of ξ_{al}



Changes the Amplitude

Never seen on CS-2

Semi-analytical models for delay/Doppler altimetry

Parameter Estimation

- Least squares estimation
- Levenberg-Marquardt algorithm
- Scenarios

  - DDA3: $\boldsymbol{\theta} = (\mathrm{SWH}, \tau, P_u)^T$ et $\xi_{\mathrm{al}} = \xi_{\mathrm{ac}} = 0°$
  - DDA4: $\boldsymbol{\theta} = (\mathrm{SWH}, \tau, P_u, \xi_{\mathrm{ac}})^T$ et $\xi_{\mathrm{al}} = 0°$
  - DDA5: $\boldsymbol{\theta} = (\mathrm{SWH}, \tau, P_u, \xi_{\mathrm{ac}}, \xi_{\mathrm{al}})^T$

ξ_{al} and ξ_{ac} can be introduced (if known from STR) as input parameters of DDA3 or DDA4

Semi-analytical models for delay/Doppler altimetry

Fit on real data (CS-2)

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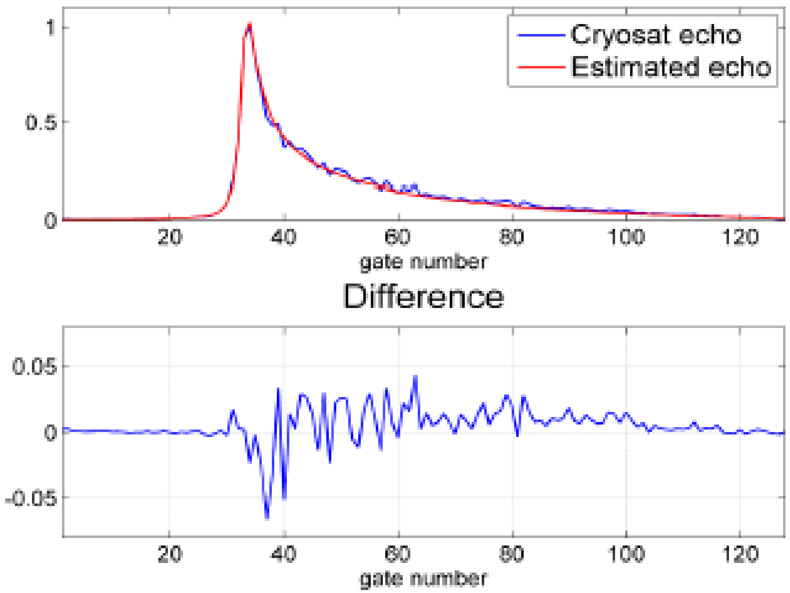
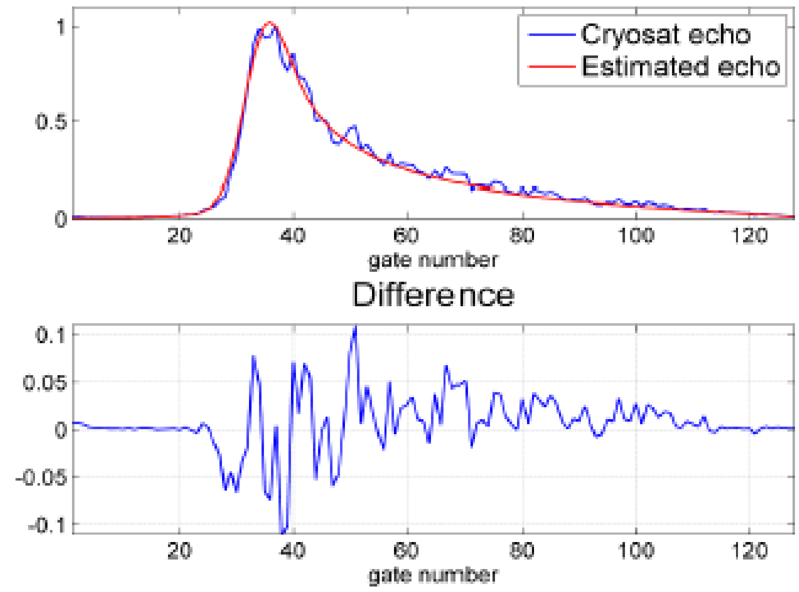


$\mathrm{SWH} = 0.57$ m and $\mathrm{NRE} = 0.07$.



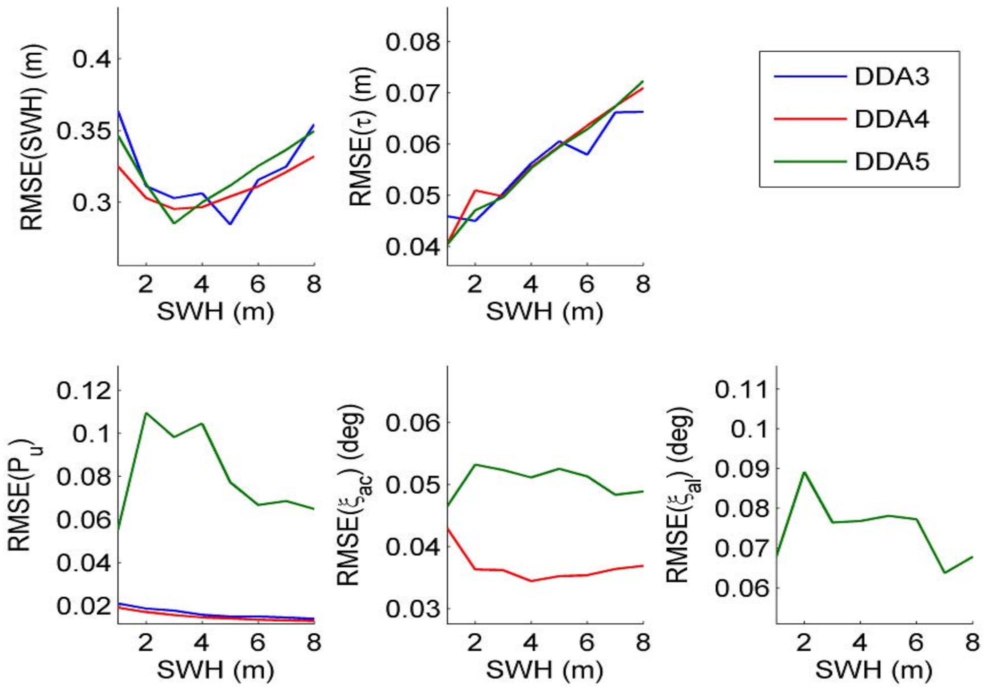
$\mathrm{SWH} = 5.84$ m and $\mathrm{NRE} = 0.102$.

NRE = Normalized Reconstruction Error

- Very good fit between CS-2 waveforms and Halimi model for all SWH
  - ❑ At the toe
  - ❑ on the leading edge
  - ❑ on the tail of the echo

Semi-analytical models for delay/Doppler altimetry

Results for simulated data



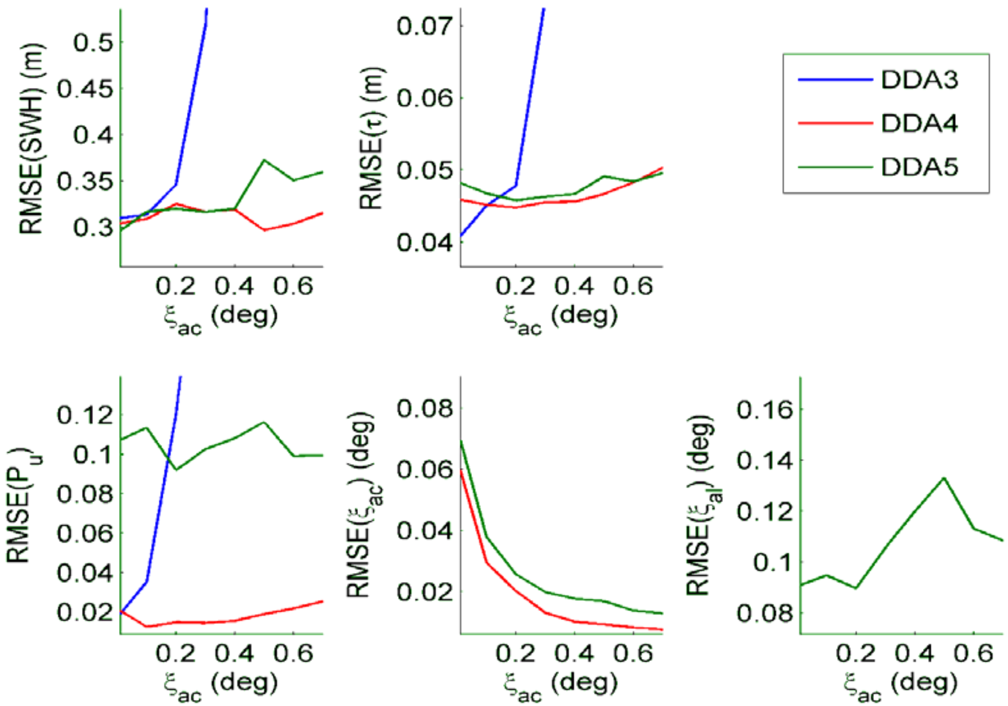
- RMSEs as a function of SWH

Parameters: $P_u = 1$, $\tau = 31$ gates , $\xi_{\text{al}} = 0°$ and $\xi_{\text{ac}} = 0°$.

- DDA3 and DDA4 are consistent and provide good performance
  DDA5 has degraded performances due to the strong Pu/ξ_{al} correlation

Semi-analytical models for delay/Doppler altimetry

Results for simulated data



- RMSEs as a function of $\xi_{\rm ac}$

Parameters: $P_u = 1$, $\text{SWH} = 2$ m, $\tau = 31$ gates and $\xi_{\rm al} = 0°$.

- Significant errors on estimations with unknown mispointing angles

Results for real data (CS-2, 400 seconds)

		τ (m)	SWH (m)	P_u	ξ_{ac} (deg)	ξ_{al} (deg)	ξ (deg)
STDs (20 Hz)	DDA3	0.0843	0.355	1.933	-	-	-
	DDA4	0.0827	0.351	1.871	0.031	-	0.031
	DDA5	0.0828	0.416	13.446	0.0413	0.0922	0.0866

Standard deviations for DDA3, DDA4 and DDA5 algorithms.

- Better results with DDA4 solution

$$\boldsymbol{\theta} = (\text{SWH}, \tau, P_u, \xi_{ac})^T \text{ et } \xi_{al} = 0°$$

- DDA5 has lower performances due to the strong Pu/ξal correlation

Conclusions & Perspectives

Conclusions

- A new semi-analytical Delay/Doppler Altimetry model has been defined and validated (DDA3 published, DDA5 to be published)
- Delay/Doppler altimetry provides increased precision than conventional altimetry
- Accounting for antenna mispointing improves the performances of the model especially for high mispointing angles
- DDA4 more robust than DDA5
- On stacked echos, along track mispointings are required to remove ambiguity with Pu

Perspectives

- To perform a full calval activity on CS-2 data and cross comparison with CPP and other solutions …
- To improve the estimation by considering the delay/doppler matrix instead of the mutli-look echo (ξ_{al} from STR or derived from the 2D delay/doppler map)