Toward global predictions of the mode-1 internal tide

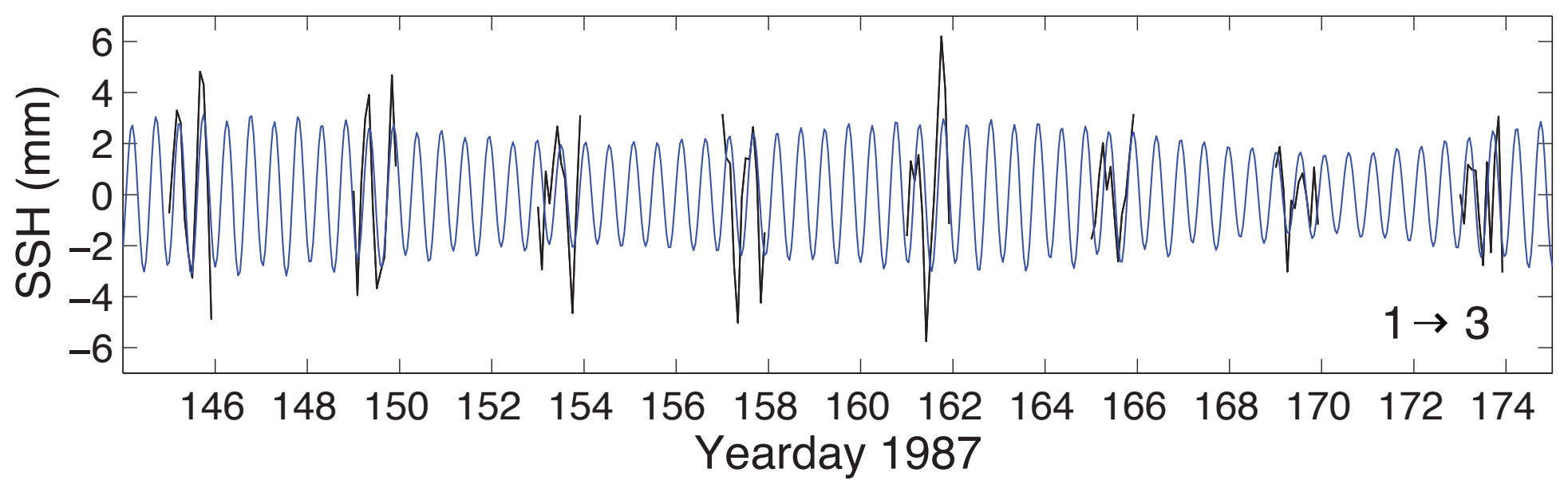
B.D. Dushaw *Applied Physics Laboratory, University of Washington, Seattle, WA USA*

ABSTRACT

A frequency–wavenumber tidal analysis for deriving internal-tide harmonic constants from TOPEX/Poseidon (T/P) measurements of sea-surface height (SSH) has been developed, taking advantage of the evident temporal and spatial coherence and the weak dissipation of internal tides. The approach is a close cousin to Fourier series or objective mapping methods for fitting and interpolating data, but employing basis functions of traveling waves. Previous analyses consisted of simple tidal analysis at individual points, with resulting harmonic constants that were inconsistent with the dispersion relation and not self-consistent at altimeter track crossover points. Such analyses have difficulty in distinguishing between the effects of interference, incoherence, and dissipation. The frequency–wavenumber analysis provides an objective way to interpolate the internal tides measured along altimetry tracks to arbitrary points, while leveraging all available data for optimal tidal estimates. Tidal analysis of T/P data from 2000 to 2007 is used to predict in situ time series measured by tomography during the 2001–2002 Hawaiian Ocean mixing experiment (HOME), the 1987 Reciprocal Tomography Experiment (RTE87), and the 1991 Acoustic Mid-Ocean Dynamics Experiment (AMODE), demonstrating both the temporal coherence and the lack of incoherent elements to this wave propagation. The temporal coherence is directly evident in time series measured by acoustic tomography. Further, after correcting for changes in background stratification, it is evident that the internal-tide waves experience little attenuation as they cross the Northe Pacific basin. A significant fraction of the variability of internal waves, that component associated with mode-1 internal tides, appears to be predictable over most of the world's oceans, using harmonic constants derived from satellite altimetry.

PROLOGUE: 1995

Acoustic transmissions made during the 1987 Reciprocal Tomography Experient (RTE87) along a 750-, 1000-, and 1250-km paths showed coherent tidal variations (Dushaw et al. 1995). These observations employed the sum of reciprocal travel times, so the variations were assocated with sound speed (temperature). The acoustic measurement kernel (rays that cycled throughout the water column) indicated the variability was associated with the mode-1 internal tide. A tidal analysis of the 120-day record length from the 750-km zonal path accounted for 56% of the variance; little incoherent tidal variability was apparent. With good temporal resolution and an inherent average over depth, the acoustic observations are complementary to SSH observations by satellite: the acoustic observations directly showed the temporal coherence that that was later inferred from SSH.



Baroclinic mode-1 variations observed along a 750-km long zonal path in the central North Pacific. An inverse was used to solve for mode amplitude from acoustic travel times, and then the equivalent sea-surface height was inferred. 30 days of the 120-day record length are shown.
Black: Observations, Blue: A tidal analysis.

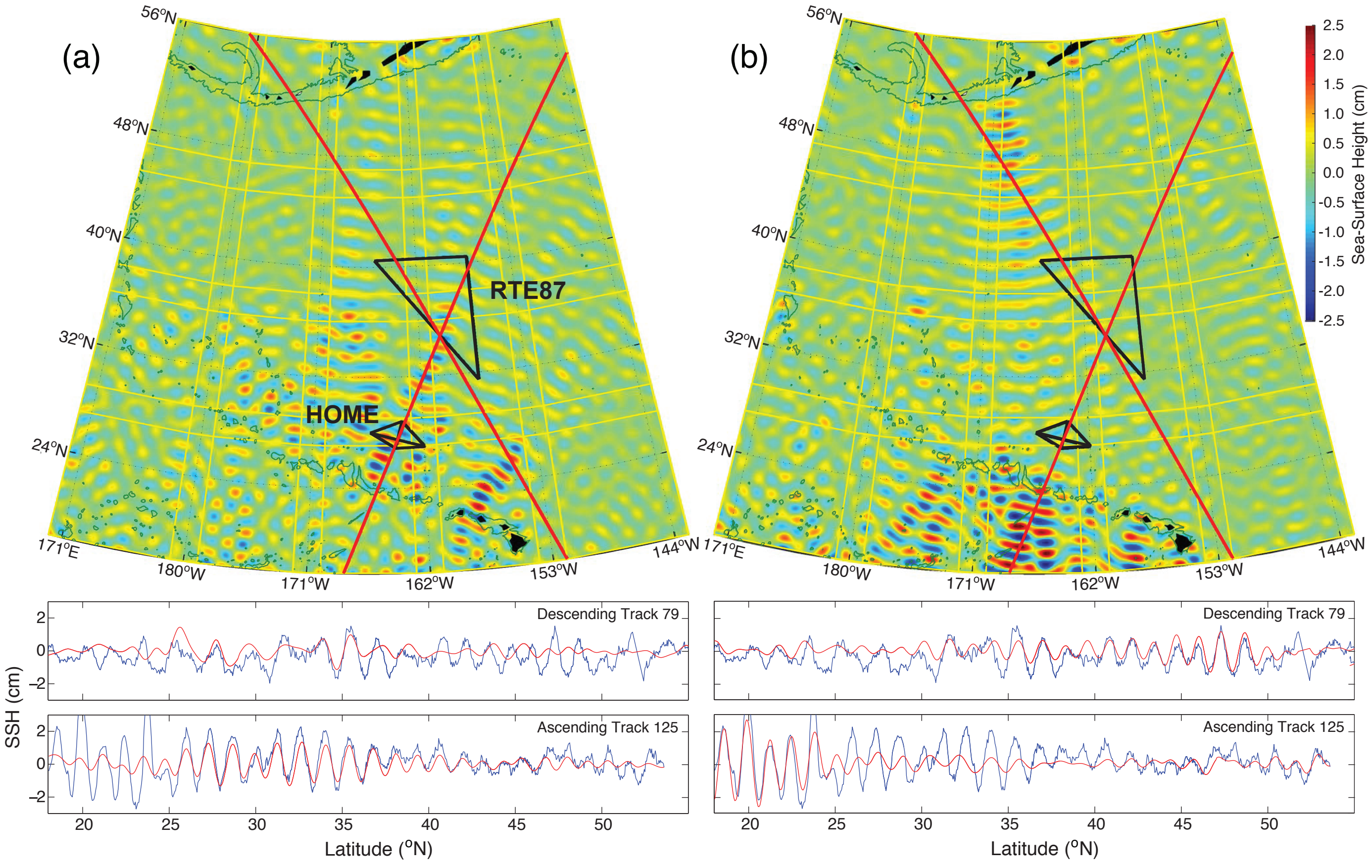
These observations, reported almost 20 years ago, foreshadow the main conclusion reported here: The mode-1 internal tide appears to be predictable over many regions of the world's oceans. These waves are large (150-km wavelength) and fast (3.5 m/s phase speed). They travel the distance from Hawaii to the Aleutians in about 12 days. Mesoscale variability, with 30-day time scales, O(100 km) spatial scales and confined to the upper ocean, has little influence on these waves.

Question: Why has it taken the better part of 20 years for the predictable nature of these waves to become apparent?
Answer: (I know the answer, but it reflects poorly on the human aspect of science...)

Example:
The scene: 1995 American Geophysical Union conference, San Francisco, in front of a poster describing AMODE internal tides.
The characters: Dushaw and an ONR Scientific Program Officer.

Dushaw: ...the internal-tide harmonic constants...
OSPO: Yes.
Dushaw: ...the phase-locked internal tide...
OSPO: Yes.
Dushaw: ...the deterministic internal tide..
OSPO: Yes.
Dushaw: ...this means some aspect of the mode-1 internal tide is predictable...
OSPO: No. [Frowns, shakes head, walks off]

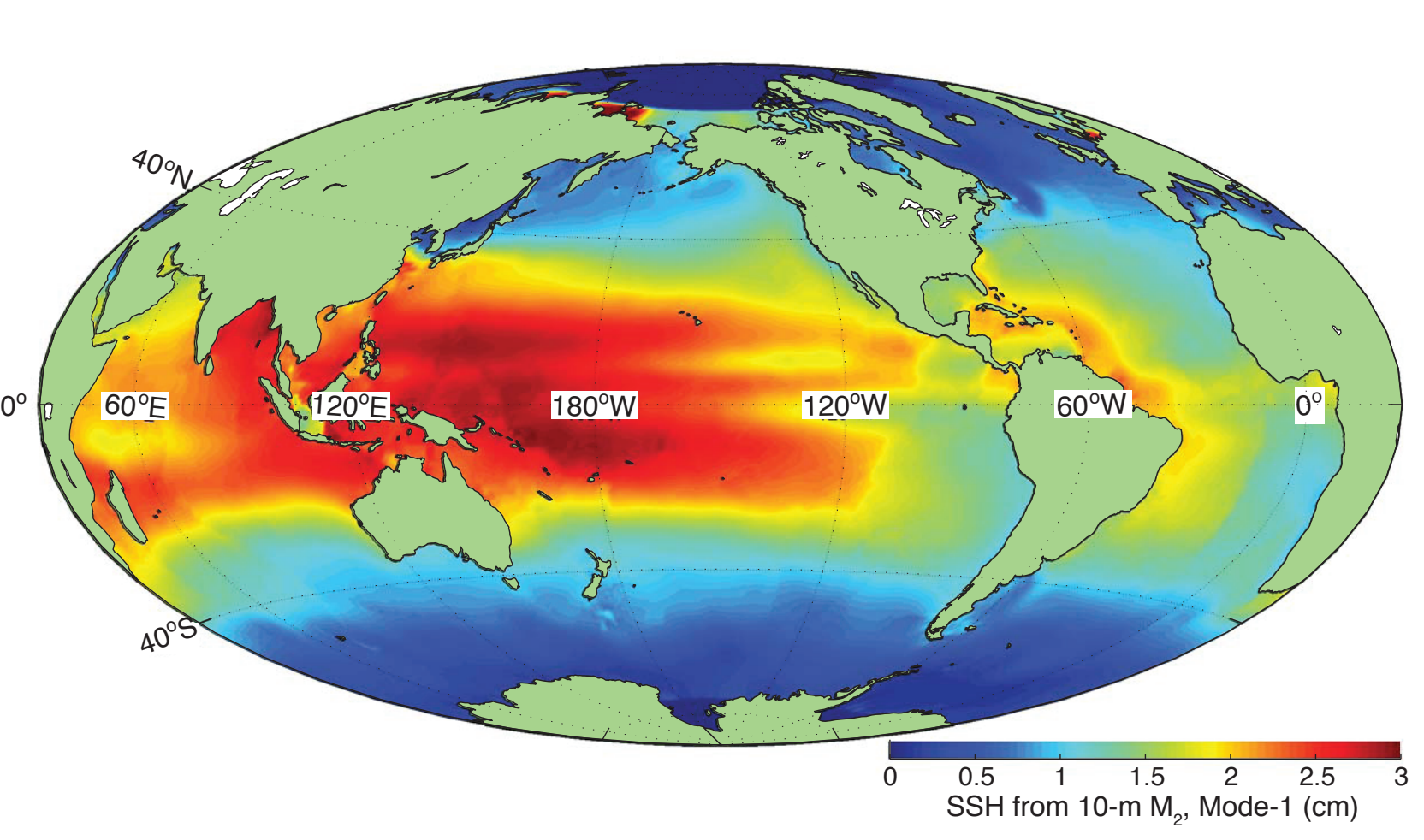
These phrases are all equivalent, however. If one speaks of "harmonic constants" one has an expectation of predictability.



Maps of the mode-1 M_2 internal tide derived from TOPEX/POSEDON altimetry SSH, assuming northward wavenumbers (left), and southward wavenumbers (right). Lower panels indicate internal tides along the red tracks derived by tidal analysis at individual points (blue) and by frequency-wavenumber tidal analysis (red). Both northward and southward wavenumbers are required to fit the data.

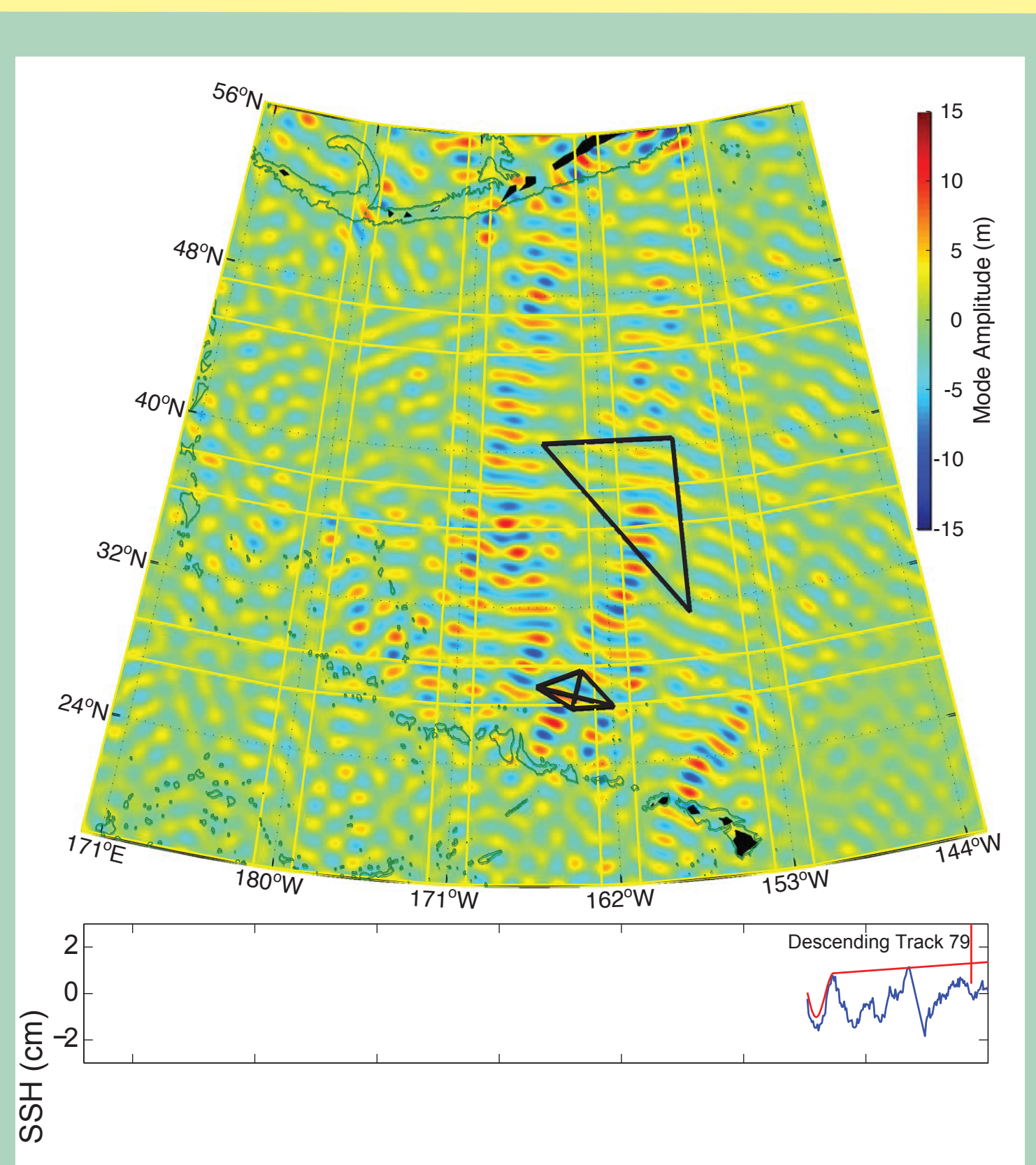
The tidal analysis is applied to the 20 square regions denoted by the yellow boxes separately, with the complete solution formed by tapering the regions together. Although the analyses within the regions are independent, the waves are seemless.

SEA SURFACE HEIGHT↔INTERNAL DISPLACEMENT



The relation of sea-surface height to internal mode amplitude is a function of stratification. Mode amplitude is closely associated with mode energy. This figure shows the sea surface expression of internal tides with 10-m internal displacement over the world's oceans. North of Hawaii, the sea-surface signal rapidly decreases. This "decay" is the result of stratification, not tidal incoherence or attenuation.

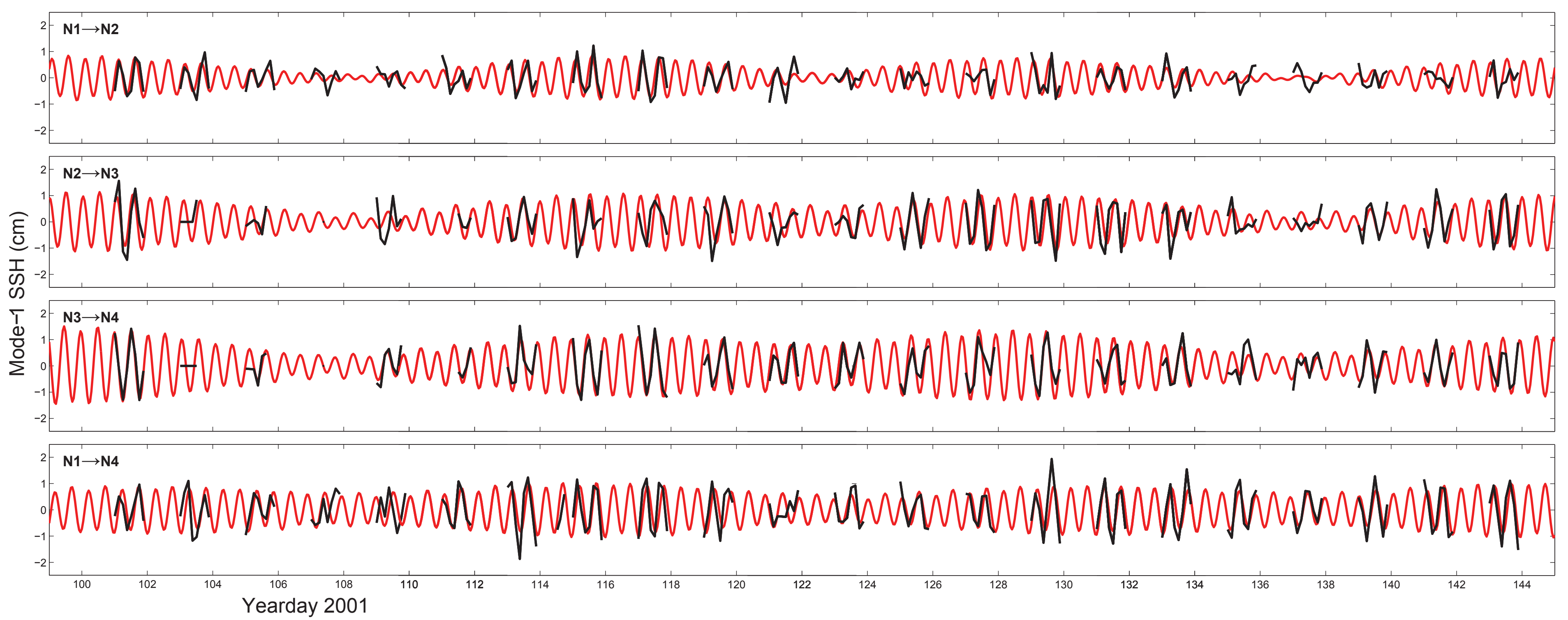
The relation between surface expression and internal displacement can be applied to the sea-surface estimates at left to obtain the estimate for mode amplitude at right, a more clear realization of what the internal tide is doing.



Results for mode amplitude for northward wavenumbers. The internal tide exhibits very little decay, consistent with the retention of coherence. The lower panels compare point-wise and frequency-wavenumber estimates (using both northward and southward wavenumbers), indicating a good fit to the SSH data.

NEXUS
(ANIMATIONS)

PREDICTIONS OF HOME TOMOGRAPHY DATA



INTRODUCTION BY WAY OF TWO ERRORS

In their paper "Mapping low-mode internal tides from multisatellite altimetry", Zhao, Alford and Girton (Oceanography 2012) summarize the present analysis as, "One proposed way forward has been to construct an inverse solution for a beam pattern based on a continuum of waves over a large area of ocean (Dushaw, 2002; Dushaw et al., 2011), but this approach ignores the spatial inhomogeneity of energy sources and sinks." This statement has two errors that will be used to introduce the analysis here.

BEAMS Although Zhao and Alford and others have interpreted these internal waves as "beams" that view is not altogether correct. From formal antenna/source theory such beams exist, of course, but the interpretations have tended to be more along the lines of something like a "laser beam" of internal tide energy. It is more correct to view this phenomena as an artifact of interference - the patterns of waves (top panels) are interference patterns, with regions constructive and destructive interference. The difference in interpretation may at first appear nuanced, but viewed as an interference pattern, the extraordinary temporal and spatial coherence should be immediately obvious. **An interference pattern is an inherently sensitive, unstable phenomena in nature, and yet the internal tide interference patterns have been observed by altimetry for decades. Any incoherent contributions to the underlying waves would render the interference patterns unstable and these waves unobservable by altimetry.**

IGNORES SOURCES AND SINKS This is incorrect, as is obvious from Dushaw et al. 2011. The internal tide field is modeled as an interference pattern using a large number of basis functions that act together to obtain a "best fit" to the data over time and space. The approach is similar to a Fourier series, but using traveling waves as basis functions. Does the statement: "A Fourier series can be used to model arbitrary functions, but it ignores sources and sinks." make any sense? The analysis of Dushaw et al. 2011 went to great lengths to test and demonstrate the ability of the least squares fit to interpolate the tidal field between the altimeter tracks, which was one of the principle aims of the project.

TIDAL MODEL

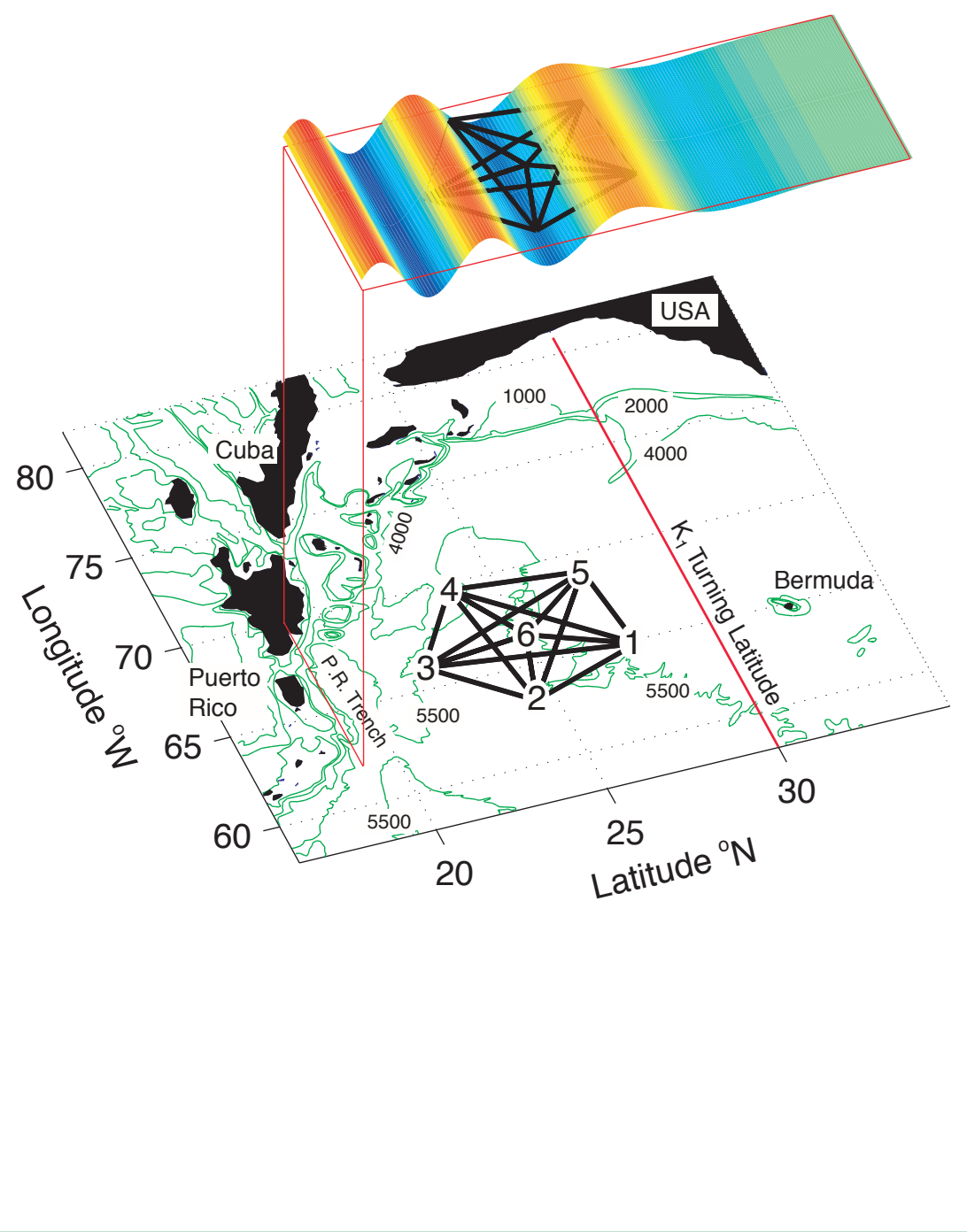
To analyze the altimeter data for the internal tide, the sea-surface height at any point and time $(\vec{x}_i, t_j)$ is modeled as a linear superposition of a large number of traveling waves:

$$\eta(\vec{x}_i,t_j)=\sum_{n,m=1}^{N,M} f_n(t_j)A_{n,m}\cos(\omega_n t_j-\vec{k}_m\cdot\vec{x}_i-G_{n,m}+(V_{0n}+u_n(t_j)))+\varepsilon$$

with frequencies ω_n and wavenumbers $\vec{k}_m$. The lunar node factor f_n, and equilibrium argument $V_{0n}+u_n$ associated with each tidal frequency are explicitly included. The analysis treats wavenumber and frequency on an equal basis. A weighted least-squares fit is made to the along-track altimeter data to solve for the amplitude and phase of each wavenumber and frequency. Eight tidal frequencies, four semidiurnal and four diurnal, were included. The wavenumbers used in the fit require careful consideration. From the least-squares solution, harmonic constants for any particular constituent at any point, or predictions for time series of sea-surface height at any point, can be derived.

The philosophy here is the same as that of an objective map or a Fourier series. A solution for a "best fit" to the data is constructed by a linear superposition of a large number of sinusoids; each individual sinusoid has little meaning without the others. The solution allows for data noise and gives an optimal estimate for interpolation of the solution away from the altimeter track lines, consistent with data, uncertainties and internal-tide kinematics. The sinusoids in this case have frequency and wavenumber consistent with the dispersion relation. This model leverages the obvious temporal and spatial coherence of the waves and excludes mesoscale variability.

INTERNAL TIDES NEAR THEIR TURNING LATITUDE



REFERENCES

Dushaw, B. D., B. D. Cornuelle, P. Worcester, B. M. Howe, and D. S. Luther, 1995: Barotropic and baroclinic tides in the central North Pacific Ocean determined from long-range reciprocal acoustic transmissions. *J. Phys. Oceanogr.*, **25**, 631-647.

Dushaw, B. D., G. D. Egbert, P. Worcester, B. Cornuelle, B. Howe, and K. Metzger, 1997: A TOPEX/POSIEDON global tidal model (TPXO.2) and barotropic tidal currents determined from long-range acoustic transmissions. *Progress in Oceanography*, **40**, 337-367.

Dushaw, B. D., 1998: Resonant diurnal internal tides in the North Atlantic. *Geophys. Res. Lett.*, **25**, 2189-2192.

Dushaw, B. D., 2002: Mapping low-mode internal tides near Hawaii using TOPEX/POSEIDON altimter data. *Geophys. Res. Lett.*, **28**, 24-26.

Dushaw, B. D., 2004: Mapping and wavenumber resolution of line-integral data for observations of low-mode internal tides. J. Ocean. Atmos. Tech., **20**, 1043-1059.

Dushaw, B. D., 2006: Mode-1 internal tides in the western North Atlantic Ocean. *Deep-Sea Res. I*, **53**, 449-473.

Dushaw, B. D., P. F. Worcester, and M. A. Dziecuich, 2011: On the predictability of mode-1 internal tides. *Deep-Sea Res. I*, **58**, 677-698.

Rainville, L., T. M. S. Johnston, G. S. Carter, M. A. Merrifield, R. Pinkel, P. F. Worcester, and B. D. Dushaw, 2009: Interference pattern and propagation of the M_2 internal tide south of the Hawaiian Ridge, *J. Phys. Oceanogr.*, **40**, doi: 10.1175/2009JPO4256.1.

Munk, W., P. Worcester, and C. Wunsch, 1995: *Ocean Acoustic Tomography*, Cambridge University Press, Cambridge.