

Incertitudes sur l'estimation de l'épaisseur de glace de mer à partir de l'altimétrie satellitaire

Atelier glaciologie et altimétrie - 25/06/2019

JC. Poisson, P. Thibaut, S.Fleury



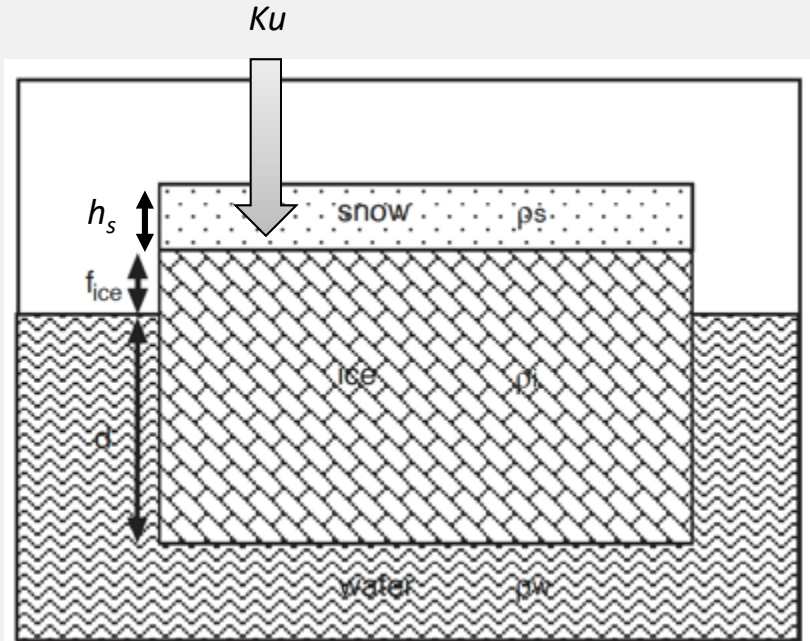
Introduction

- For many years, the Sea Ice Thickness computed from altimeter measurement is derived from the freeboard measurement thanks to the Hydrostatic Equilibrium equation
- SIT uncertainty is hard to estimate from measurements since large scale validation is not easy (there is no other Pan-Arctic/Antarctic ice thickness measurements, only local validation can be made thanks to in-situ measurements or airborne campaign)
- In the frame of the ESA POLARICE project, a review of the ice thickness uncertainty has been done based on literature in order to prepare the CRISTAL mission
- We propose here a summary of the uncertainty analysis on the SIT derived from satellite altimetry.



Hydrostatic Equilibrium (Ku-band)

HYPOTHESIS: The Ku waveform is reflected at the snow/ice interface



Water Density

Ice Freeboard

Snow Density

$$SIT = \frac{\rho_w}{\rho_w - \rho_{ice}} f_{ice} + \frac{\rho_s}{\rho_w - \rho_{ice}} h_s$$

Sea Ice Thickness

Ice Density

Snow Depth



Accounting for snow delay

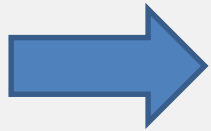
- Range delay due to the snow : $f_{ice} = f_{radar} + (1 - \frac{c_s}{c})h_s$

- In literature :

Ulaby et al. 1986 → $\frac{c_s}{c} = (1 + 0.51\rho_s/1000)^{-1.5}$

with $\rho_s = 290 \text{ kg/m}^3$

Tiuri et al. 1984 → $\frac{c_s}{c} = \frac{1}{\sqrt{1 + 1.7\rho_s/1000 + 0.7(\rho_s/1000)^2}}$

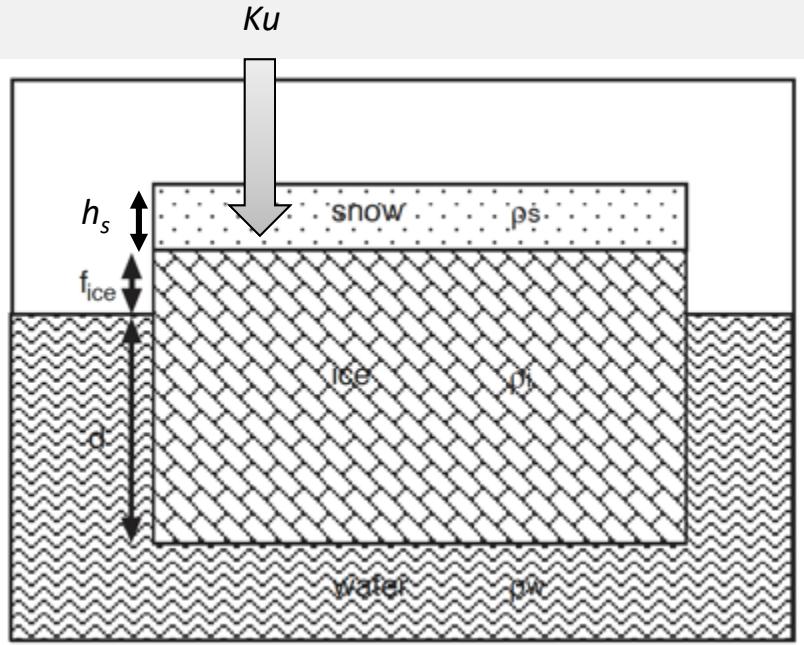


$$SIT = \frac{\rho_w}{\rho_w - \rho_{ice}} f_{radar} + \frac{(1 - c_s/c)\rho_w + \rho_s}{\rho_w - \rho_{ice}} h_s$$



Hydrostatic Equilibrium (Ku-band)

HYPOTHESIS: The Ku waveform is reflected at the snow/ice interface



$$SIT = \frac{\rho_w}{\rho_w - \rho_{ice}} f_{radar} + \frac{(1 - c_s/c) \rho_w + \rho_s}{\rho_w - \rho_{ice}} h_s$$

Diagram illustrating the components of the equation:

- Water Density (ρ_w)
- Freeboard (f_{radar})
- Snow Density (ρ_s)
- Ice Density (ρ_{ice})
- Velocity ratio (c_s/c)
- Snow Depth (h_s)



Uncertainty sources on SIT

$\varepsilon_{SIT}^2 =$

$$\varepsilon_{f_{ice}}^2 \left(\frac{\rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{h_s}^2 \left(\frac{-\frac{\rho_w}{(1 + 0.00051\rho_s)^{1.5}} + \rho_s + \rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{\rho_s}^2 \left(\frac{h_s \left(\frac{0.000765\rho_w}{(1 + 0.00051\rho_s)^{2.5}} + 1 \right)}{\rho_w - \rho_{ice}} \right)^2$$

$$+ \varepsilon_{\rho_w}^2 \left(\frac{\rho_{ice}f_{ice} + h_s \left(\rho_s - \frac{\rho_{ice}}{(1 + 0.00051\rho_s)^{1.5}} + \rho_{ice} \right)}{(\rho_w - \rho_{ice})^2} \right)^2 + \varepsilon_{\rho_{ice}}^2 \left(\frac{f_{ice}\rho_w + h_s \left(-\frac{\rho_w}{(1 + 0.00051\rho_s)^{1.5}} + \rho_s + \rho_w \right)}{(\rho_w - \rho_{ice})^2} \right)^2$$

Based on CS-2 performances

MYI

$$\varepsilon_{SIT}^2 = \varepsilon_{f_{ice}}^2 52.002 + \varepsilon_{h_s}^2 11.492 + \varepsilon_{\rho_s}^2 1.554e^{-5}$$

$$+ \varepsilon_{\rho_w}^2 2.657e^{-4} + \varepsilon_{\rho_{ice}}^2 3.273e^{-4}$$

FYI

$$\varepsilon_{SIT}^2 = \varepsilon_{f_{ice}}^2 91.075 + \varepsilon_{h_s}^2 20.127 + \varepsilon_{\rho_s}^2 5.376e^{-6}$$

$$+ \varepsilon_{\rho_w}^2 1.758e^{-4} + \varepsilon_{\rho_{ice}}^2 2.056e^{-4}$$

In the CryoSat-2 SIT error budget, only one error source comes from the altimeter measurement, all the others come from external measurements / models.



Uncertainty sources on SIT

$\varepsilon_{SIT}^2 =$

$$\varepsilon_{f_{ice}}^2 \left(\frac{\rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{h_s}^2 \left(\frac{-\frac{\rho_w}{(1 + 0.00051\rho_s)^{1.5}} + \rho_s + \rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{\rho_s}^2 \left(\frac{h_s \left(\frac{0.000765\rho_w}{(1 + 0.00051\rho_s)^{2.5}} + 1 \right)}{\rho_w - \rho_{ice}} \right)^2$$

$$+ \varepsilon_{\rho_w}^2 \left(\frac{\rho_{ice}f_{ice} + h_s \left(\rho_s - \frac{\rho_{ice}}{(1 + 0.00051\rho_s)^{1.5}} + \rho_{ice} \right)}{(\rho_w - \rho_{ice})^2} \right)^2 + \varepsilon_{\rho_{ice}}^2 \left(\frac{f_{ice}\rho_w + h_s \left(-\frac{\rho_w}{(1 + 0.00051\rho_s)^{1.5}} + \rho_s + \rho_w \right)}{(\rho_w - \rho_{ice})^2} \right)^2$$

Based on CS-2 performances

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$$\varepsilon_{SIT}^2 = \varepsilon_{f_{ice}}^2 52.002 + \varepsilon_{h_s}^2 11.492 + \varepsilon_{\rho_s}^2 1.554e^{-5}$$

$$+ \varepsilon_{\rho_w}^2 2.657e^{-4} + \varepsilon_{\rho_{ice}}^2 3.273e^{-4}$$

$\rho_{ice} = 882 \text{ kg/m}^3$ Alexandrov et al. 2010

$\rho_w = 1024 \text{ kg/m}^3$ Wadhams et al. 1992

$\rho_s = 290 \text{ kg/m}^3$ Warren et al. 1999

FYI

$$\varepsilon_{SIT}^2 = \varepsilon_{f_{ice}}^2 91.075 + \varepsilon_{h_s}^2 20.127 + \varepsilon_{\rho_s}^2 5.376e^{-6}$$

$$+ \varepsilon_{\rho_w}^2 1.758e^{-4} + \varepsilon_{\rho_{ice}}^2 2.056e^{-4}$$

$\rho_{ice} = 916.7 \text{ kg/m}^3$ Alexandrov et al. 2010

$\rho_w = 1024 \text{ kg/m}^3$ Wadhams et al. 1992

$\rho_s = 290 \text{ kg/m}^3$ Warren et al. 1999

(varies between 240 and 340 kg/m³)



Average errors on Sea Ice Thickness

Currently (CryoSat-2)

MYI

$$\varepsilon_{f_{ice}} = 0.03 \text{ m} \quad \text{Ricker et al. 2014}$$

$$\varepsilon_{\rho_{ice}} = 23 \text{ kg/m}^3 \quad \text{Alexandrov et al. 2010}$$

$$\varepsilon_{h_s} = 0.09 \text{ m} \quad \text{Warren et al. 1999}$$

$$\varepsilon_{\rho_s} = 3.2 \text{ kg/m}^3 \quad \text{Warren et al. 1999}$$

$$\varepsilon_{\rho_w} = 0.5 \text{ kg/m}^3 \quad \text{Wadhams et al. 1992}$$

$$f_{ice} = 0.187 \text{ m} \quad \text{Ricker et al. 2014}$$

(March 2013 TFMRA50)

$$h_s = 0.36 \text{ m} \quad \text{Warren et al. 1999}$$

(March)

FYI

$$\varepsilon_{f_{ice}} = 0.03 \text{ m} \quad \text{Ricker et al. 2014}$$

$$\varepsilon_{\rho_{ice}} = 35.7 \text{ kg/m}^3 \quad \text{Alexandrov et al. 2010}$$

$$\varepsilon_{h_s} = 0.09 \text{ m} \quad \text{Warren et al. 1999}$$

$$\varepsilon_{\rho_s} = 3.2 \text{ kg/m}^3 \quad \text{Warren et al. 1999}$$

$$\varepsilon_{\rho_w} = 0.5 \text{ kg/m}^3 \quad \text{Wadhams et al. 1992}$$

$$f_{ice} = 0.086 \text{ m} \quad \text{Ricker et al. 2014}$$

(March 2013 TFMRA50)

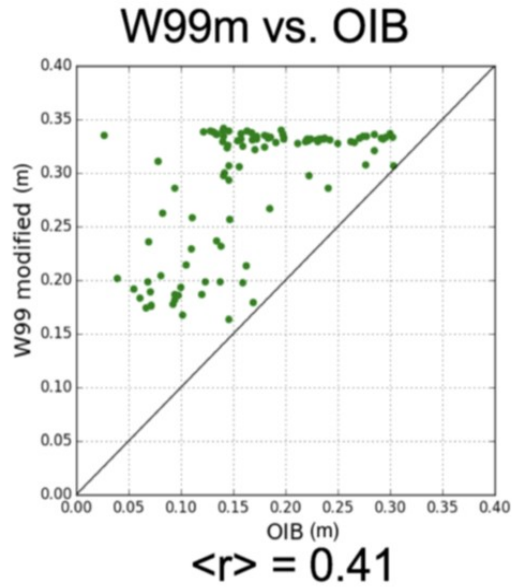
$$h_s = 0.16 \text{ m} \quad \text{Warren et al. 1999}$$

(March ÷ 2 → modified version)



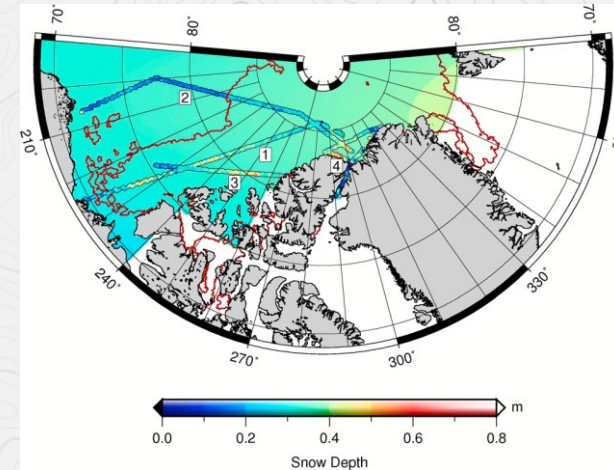
Snow depth impact

- Based on CS-2 performances
- $\mathcal{E}_{hs} = 9$ cm from Warren is largely too optimistic. A value of 15 cm seems more realistic



The comparison of the Warren99 snow depth climatology with Operation Ice Bridge (OIB) snow depth shows important discrepancies

Courtesy of S. Fleury



From Kurtz & Farrell
2011



Uncertainty on SIT (Ku-band)

Currently (CryoSat-2)

MYI

$$\varepsilon_{SIT}^2 = 0.0468 + 0.25857 + 1.59e^{-4} + 6.642e^{-5}$$

freeboard
Snow depth
Snow density
Water density

$\varepsilon_{SIT} = 0.69 \text{ m}$

Ice density

FYI

$$\varepsilon_{SIT}^2 = 0.0820 + 0.453 + 5.504e^{-5} + 4.395e^{-5}$$

freeboard
Snow depth
Snow density
Water density

$\varepsilon_{SIT} = 0.89 \text{ m}$

Ice density

➔ The snow depth is the main contributor to the SIT error budget

- Uncertainty on FYI > MYI due to the ice density uncertainty which is largely higher for FYI than for MYI
- Ice density uncertainty is the second most important contributor to the SIT error budget



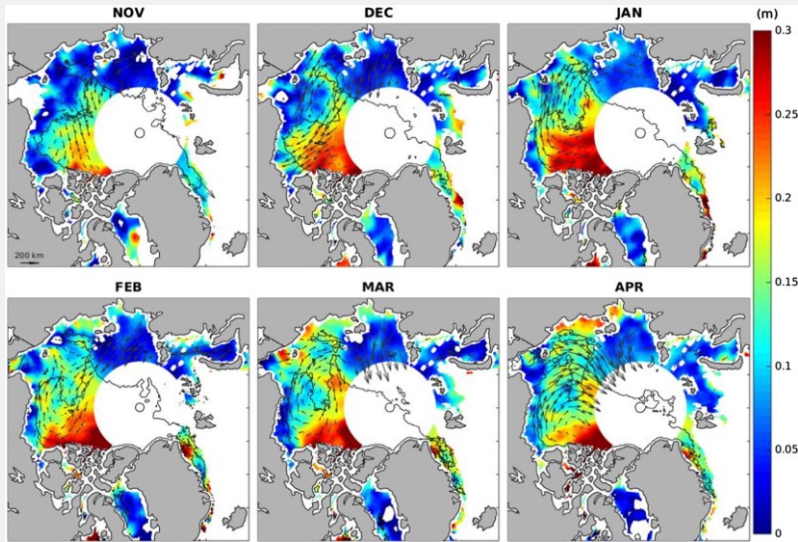
Improvements brought by CRISTAL

- Dual frequency (Ku/Ka) → Potential estimation of the snow depth by difference of Ku and Ka ranges
- Higher Ku bandwidth (500 MHz) → Better range accuracy
- SAR mode for the first time in Ka band → Better resolution and range accuracy
- Interleaved mode → Potential exploitation of Fully Focused SAR processing
- SARin Ku → Better exploitation of off-nadir returns
- Potential improvements of ground processing (Physical retrackers and freeboard computation/interpolation)

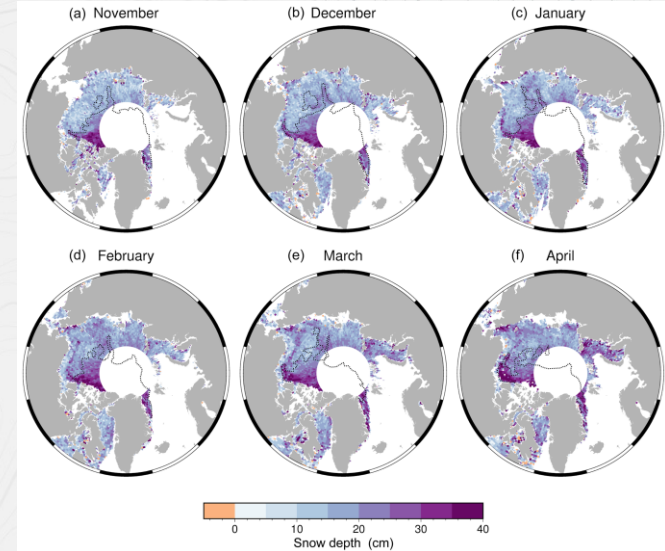


Ku/Ka Snow Depth: Firsts results/products

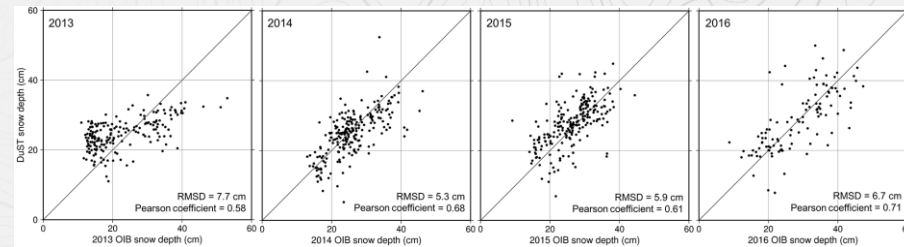
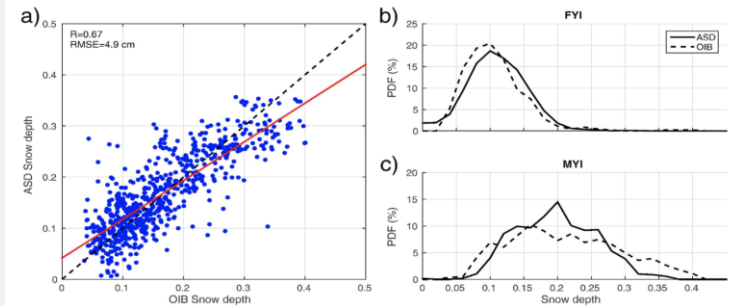
Guerreiro et al. 2016



Lawrence et al. 2018



Snow Depth
computed using
AltiKa and
CryoSat-2
measurements
at crossovers



Reducing the SIT uncertainty with CRISTAL

- Accounting for CRISTAL larger bandwidth
- \mathcal{E}_{hs} can be improved using the Ku/Ka differences for snow depth estimation
 - 6.5 cm (Lawrence et al 2018)
 - 5.0 cm (Guerreiro et al. 2016)

freeboard MYI Snow density Water density

$$\varepsilon_{SIT}^2 = 0.019 + 0.048 + 1.59e^{-4} + 6.642e^{-5}$$

 +0.173

 Ice density

$$\varepsilon_{SIT} = 0.49 \text{ m (Lawrence's snow)}$$

$$\varepsilon_{SIT} = 0.47 \text{ m (Guerreiro's snow)}$$

freeboard FYI Snow density Water density

$$\varepsilon_{SIT}^2 = 0.034 + 0.085 + 5.504e^{-5} + 4.395e^{-5}$$

 +0.262

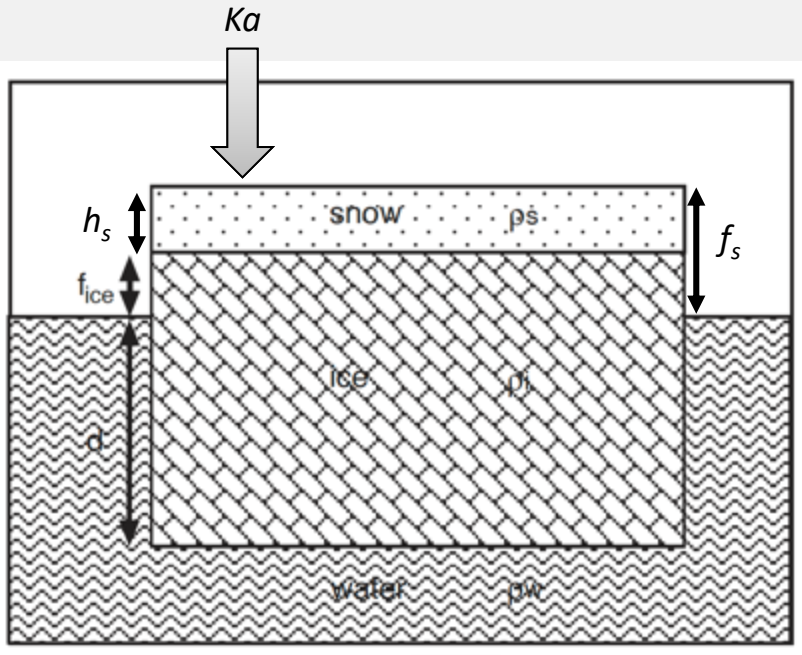
 Ice density

$$\varepsilon_{SIT} = 0.62 \text{ m (Lawrence's snow)}$$

$$\varepsilon_{SIT} = 0.59 \text{ m (Guerreiro's snow)}$$



Hydrostatic Equilibrium (Ka-Band/LRM)



Hypothesis:

Ka-band is reflected by the air/snow interface

$$SIT = \frac{\rho_w}{\rho_w - \rho_{ice}} f_s - \frac{\rho_w - \rho_s}{\rho_w - \rho_{ice}} h_s$$

The equation is annotated with boxes and arrows:

- Water Density** (blue box) points to ρ_w in the numerator of the first fraction.
- Snow Freeboard** (blue box) points to f_s .
- Snow Density** (white box) points to ρ_s in the numerator of the second fraction.
- Sea Ice Thickness** (blue box) points to f_s .
- Ice Density** (blue box) points to ρ_{ice} in the denominator of both fractions.
- Snow Depth** (white box) points to h_s .



Error on Sea Ice Thickness (Ka-Band/LRM)

$$\begin{aligned} \varepsilon_{SIT}^2 = & \varepsilon_{f_s}^2 \left(\frac{\rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{h_s}^2 \left(\frac{\rho_s}{\rho_w - \rho_{ice}} - \frac{\rho_w}{\rho_w - \rho_{ice}} \right)^2 + \varepsilon_{\rho_s}^2 \left(\frac{h_s}{\rho_w - \rho_{ice}} \right)^2 \\ & + \varepsilon_{\rho_w}^2 \left(\frac{f_s}{\rho_w - \rho_{ice}} - \frac{h_s}{\rho_w - \rho_{ice}} - \frac{f_s \rho_w}{(\rho_w - \rho_{ice})^2} - \frac{h_s \rho_s}{(\rho_w - \rho_{ice})^2} + \frac{h_s \rho_w}{(\rho_w - \rho_{ice})^2} \right)^2 \\ & + \varepsilon_{\rho_{ice}}^2 \left(\frac{f_s \rho_w}{(\rho_w - \rho_{ice})^2} + \frac{h_s \rho_s}{(\rho_w - \rho_{ice})^2} - \frac{h_s \rho_w}{(\rho_w - \rho_{ice})^2} \right)^2 \end{aligned}$$

Giles et al. 2007

MYI

$\rho_{ice} = 882 \text{ kg/m}^3$ Alexandrov et al. 2010
 $\rho_w = 1024 \text{ kg/m}^3$ Wadhams et al. 1992
 $\rho_s = 290 \text{ kg/m}^3$ Warren et al. 1999

FYI

$\rho_{ice} = 916.7 \text{ kg/m}^3$ Alexandrov et al. 2010
 $\rho_w = 1024 \text{ kg/m}^3$ Wadhams et al. 1992
 $\rho_s = 290 \text{ kg/m}^3$ Warren et al. 1999
 (ρ_s varies between 240 and 340 kg/m^3)



Average Error on Sea Ice Thickness (Ka-Band)

Currently (AltiKa LRM)

MYI

$\varepsilon_{f_s} = 0.015 \text{ m}$	Bandwidth ratio with CS-2 and 40Hz/20Hz ratio
$\varepsilon_{\rho_{ice}} = 23 \text{ kg/m}^3$	Alexandrov et al. 2010
$\varepsilon_{h_s} = 0.09 \text{ m}$	Warren et al. 1999
$\varepsilon_{\rho_s} = 3.2 \text{ kg/m}^3$	Warren et al. 1999
$\varepsilon_{\rho_w} = 0.5 \text{ kg/m}^3$	Wadhams et al. 1992
$f_s = 0.547 \text{ m}$	$f_s = f_{ice} + h_s$
$h_s = 0.36 \text{ m}$	Warren et al. 1999 (March)

FYI

$\varepsilon_{f_s} = 0.015 \text{ m}$	Bandwidth ratio with CS-2 and 40Hz/20Hz ratio
$\varepsilon_{\rho_{ice}} = 35.7 \text{ kg/m}^3$	Alexandrov et al. 2010
$\varepsilon_{h_s} = 0.09 \text{ m}$	Warren et al. 1999
$\varepsilon_{\rho_s} = 3.2 \text{ kg/m}^3$	Warren et al. 1999
$\varepsilon_{\rho_w} = 0.5 \text{ kg/m}^3$	Wadhams et al. 1992
$f_s = 0.246 \text{ m}$	$f_s = f_{ice} + h_s$
$h_s = 0.16 \text{ m}$	Warren et al. 1999 (March $\div 2 \rightarrow$ modified version)



Error on Sea Ice Thickness (Ka-Band)

Currently (AltiKa LRM)

MYI

FYI

$$\begin{aligned}\varepsilon_{SIT}^2 = & \varepsilon_{f_s}^2 52.002 + \varepsilon_{h_s}^2 26.718 + \varepsilon_{\rho_s}^2 6.427e^{-6} \\ & + \varepsilon_{\rho_w}^2 1.784e^{-4} \\ & + \varepsilon_{\rho_{ice}}^2 2.153e^{-4}\end{aligned}$$

$$\begin{aligned}\varepsilon_{SIT}^2 = & 0.0117 + 0.2164 + 6.581e^{-5} \\ & + 4.460e^{-5} \\ & + 0.114\end{aligned}$$

$$\begin{aligned}\varepsilon_{SIT}^2 = & \varepsilon_{f_s}^2 91.075 + \varepsilon_{h_s}^2 46.794 + \varepsilon_{\rho_s}^2 2.224e^{-6} \\ & + \varepsilon_{\rho_w}^2 1.183e^{-4} \\ & + \varepsilon_{\rho_{ice}}^2 1.364e^{-4}\end{aligned}$$

$$\begin{aligned}\varepsilon_{SIT}^2 = & 0.0205 + 0.379 + 2.277e^{-5} \\ & + 2.958e^{-5} \\ & + 0.174\end{aligned}$$

The impact of snow depth uncertainty is higher for Ka band than for Ku band



Conclusions

- SIT is computed from freeboard estimates and the Hydrostatic Equilibrium
- The Error budget derived from the Hydrostatic Equilibrium depends on many external error sources: the altimeter is only measuring the freeboard
- The main contributor to the SIT error budget is the snow depth uncertainty
- Ku/Ka combination allows to dramatically decrease the snow depth uncertainty
➔ Decrease of the SIT uncertainty
- A Ku/Ka mission is mandatory if we want to reduce the SIT uncertainty even if a target of 0.1 m is potentially too optimistic
- Ice density uncertainty is a strong contributor to the SIT error budget and needs to be investigated



Thank you for your attention!

